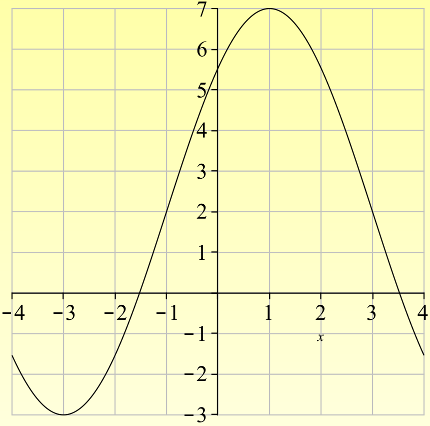
In Class Work

1. Sketch the graph of the derivative of the function whose graph is shown.

2. Use the limit definition of the derivative to find f':
   (a) $f(x) = 2x$
   (b) $f(x) = x^2$
   (c) $f(x) = 5$
   (d) $f(x) = x^3$
   (e) $f(x) = x^{-1} = \dfrac{1}{x}$

3. Find the equation of the line tangent to $y = x^3$ at $x = -2$



Hints:

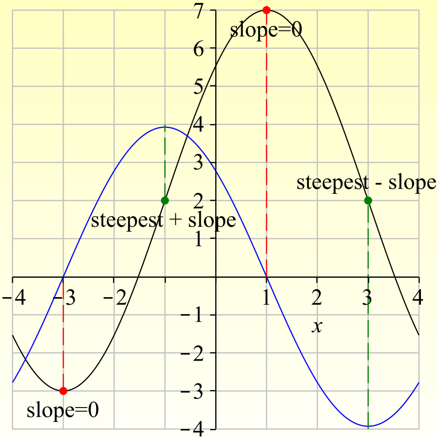
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- Remember common denominators

Solutions

1. Sketch the graph of the derivative of the function whose graph is shown.

   The slope is:

   - negative; going from very negative toward 0 on $[-4, -3]$
   - positive; increasing on $[-3, -1]$
   - positive; decreasing toward 0 on $[-1, 1]$
   - negative; going from 0 toward very negative on $[1, 3]$
   - negative; heading back toward 0 (but not reaching it) on $[3, 4]$



Solutions

2. Use the limit definition of the derivative to find the derivative of the following:

   (a) $f(x) = 2x$

$$
\begin{aligned}
m_{tan} &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{2(x+h) - 2x}{h} \\
&= \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 \\
&= 2
\end{aligned}
$$

Thus the slope of the tangent line is always 2, no matter what x we're looking at.

In other words, the graph of $2x$ has a constant slope of 2.

Question: What does having a constant slope tell us about the graph of $f(x)$?

Solutions

2. Use the limit definition of the derivative to find the derivative of the following:

   (b) $f(x) = x^2$

$$\begin{aligned} m_{tan} &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} \\ &= \lim_{h\to 0} (2x+h) \\ &= 2x \end{aligned}$$

So now we know that the graph of x^2 has a slope of 4 at $x = 2$, a slope of 2 at $x = 1$, a slope of -2 at $x = -1$, etc.

Solutions

2. Use the limit definition of the derivative to find the derivative of the following:

   (c) $f(x) = 5$

$$\begin{aligned} m_{tan} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(5) - (5)}{h} \\ &= \lim_{h \to 0} 0 \\ &= 0 \end{aligned}$$

Solutions

2. Use the limit definition of the derivative to find the derivative of the following:
   (a) $f(x) = x^3$
   Time-saver: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} \\
&= \lim_{h\to 0} \frac{\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - x^3}{h} \\
&= \lim_{h\to 0} \left(3x^2 + 3xh + h^2\right) \\
&= 3x^2
\end{aligned}$$

Solutions

2. (b) $f(x) = x^{-1} = \dfrac{1}{x}$

Remember common denominators

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{x+h} - \frac{1}{x}\right] \\
&= \lim_{h\to 0} \frac{1}{h}\,\frac{x-(x+h)}{x(x+h)} \\
&= \lim_{h\to 0} \frac{-h}{xh(x+h)} \\
&= \lim_{h\to 0} \frac{-1}{x(x+h)} \\
&= -\frac{1}{x^2} \\
&= -x^{-2}
\end{aligned}$$

3. Find the equation of the line tangent to $y = x^3$ at $x = -2$

Need: point, slope

Slope: Since $f'(x) = 3x^2$, $m_{tan} = f'(-2) = 3(-2)^2 = 12$

Point: point of tangency is $\big(-2, f(-2)\big) = \big(-2, (-2)^3\big) = (-2, -8)$.

Tangent line:

$$y + 8 = 12(x + 2) \Longrightarrow y = 12x + 16.$$

Summary: Derivatives Found Using Limit Definition

$$
\begin{aligned}
f(x) = 5 &\implies f'(x) = 0 \text{ (Fri 9/24)} \\
f(x) = 2x &\implies f'(x) = 2 \text{ (Fri 9/24)} \\
f(x) = x^2 &\implies f'(x) = 2x^1 \text{ (Fri 9/24)} \\
f(x) = x^3 &\implies f'(x) = 3x^2 \text{ (Fri 9/24)} \\
f(x) = x^{-1} &\implies f'(x) = -x^{-2} \text{ (Fri 9/24)} \\
f(x) = x^{1/2} &\implies f'(x) = \frac{1}{2}x^{-1/2} \text{ (Reading for Fri)} \\
f(x) = x^2 - 3x &\implies f'(x) = 2x - 3 \text{ (Th 9/23)} \\
f(x) = 2x^2 - 3x &\implies f'(x) = 4x - 3 \text{ (RQ for Fri 9/24)} \\
f(x) = 3x^3 + 2x - 1 &\implies f'(x = 9x^2 + 2 \text{ (Reading for F)}
\end{aligned}
$$

Summary: Derivatives Found Using Limit Definition

$$
\begin{aligned}
f(x) = 5x^0 &\implies f'(x) = 0 \text{ (Fri 9/24)} \\
f(x) = 2x^1 &\implies f'(x) = 2(1) \text{ (Fri 9/24)} \\
f(x) = x^2 &\implies f'(x) = 2x^1 \text{ (Fri 9/24)} \\
f(x) = x^3 &\implies f'(x) = 3x^2 \text{ (Fri 9/24)} \\
f(x) = x^{-1} &\implies f'(x) = -1x^{-2} \text{ (Fri 9/24)} \\
f(x) = x^{1/2} &\implies f'(x) = \frac{1}{2}x^{-1/2} \text{ (Reading for Fri)} \\
f(x) = x^2 - 3x^1 &\implies f'(x) = 2x - 3(1) \text{ (Th 9/23)} \\
f(x) = 2x^2 - 3x^1 &\implies f'(x) = 2(2)x^1 - 3(1) \text{ (RQ for Fri 9/24)} \\
f(x) = 3x^3 + 2x^1 - 1(x^0) &\implies f'(x) = 3(3)x^2 + 2(1) + 0 \text{ (Reading for F)}
\end{aligned}
$$

Summary: Derivatives Found Using Limit Definition

$$
\begin{aligned}
f(x) = 5x^0 &\implies f'(x) = 0 \text{ (Fri 9/24)} \\
f(x) = 2x^1 &\implies f'(x) = 2(1) \text{ (Fri 9/24)} \\
f(x) = x^2 &\implies f'(x) = 2x^1 \text{ (Fri 9/24)} \\
f(x) = x^3 &\implies f'(x) = 3x^2 \text{ (Fri 9/24)} \\
f(x) = x^{-1} &\implies f'(x) = -1x^{-2} \text{ (Fri 9/24)} \\
f(x) = x^{1/2} &\implies f'(x) = \frac{1}{2}x^{-1/2} \text{ (Reading for Fri)} \\
f(x) = x^2 - 3x^1 &\implies f'(x) = 2x - 3(1) \text{ (Th 9/23)} \\
f(x) = 2x^2 - 3x^1 &\implies f'(x) = 2(2)x^1 - 3(1) \text{ (RQ for Fri 9/24)} \\
f(x) = 3x^3 + 2x^1 - 1(x^0) &\implies f'(x) = 3(3)x^2 + 2(1) + 0 \text{ (Reading for F)}
\end{aligned}
$$

Conjectures: Looks as if $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$,

$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$, and $\frac{d}{dx}\left(cf(x)\right) = cf'(x)$

Remember: Pascal's Triangle

$(x+h)^0$
$(x+h)^1$
$(x+h)^2$
$(x+h)^3$
$(x+h)^4$
$(x+h)^5$

Remember: Pascal's Triangle

$(x+h)^0$	1
$(x+h)^1$	$x+h$
$(x+h)^2$	$x^2+2xh+h^2$
$(x+h)^3$	$x^3+3x^2h+3xh^2+h^3$
$(x+h)^4$	$x^4+4x^3h+6x^2h^2+4xh^3+h^4$
$(x+h)^5$	

Remember: Pascal's Triangle

$(x+h)^0$	1	1
$(x+h)^1$	$x+h$	1 1
$(x+h)^2$	$x^2+2xh+h^2$	1 2 1
$(x+h)^3$	$x^3+3x^2h+3xh^2+h^3$	1 3 3 1
$(x+h)^4$	$x^4+4x^3h+6x^2h^2+4xh^3+h^4$	1 4 6 4 1
$(x+h)^5$		1 5 10 10 5 1