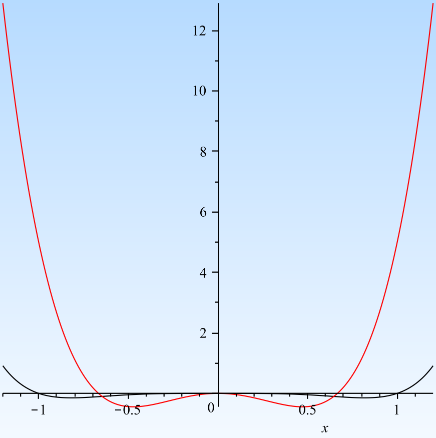
Types of Functions We Can't Yet Differentiate

- $f(x) = (x^6 - 14x^5 + 27x^{-3} - 13)(101x^{-1} + 14x^6 + 13 - 42\sqrt{x})$
- $g(x) = \dfrac{x^7 - \sqrt{x}}{14x^2 + 12}$
- $h(x) = \left(x^2 + 13x - \frac{2}{x}\right)^{1/3}$
- $j(x) = \cos(x^2)$
- $k(x) = \sin(e^{14x})$
- $m(x) = \ln(\sqrt{x} - 14)$

Consider $f(x) = (x^3 - x^2)(x^3 + x^2)$.
Is $f'(x) = (3x^2 - 2x)(3x^2 + 2x)$?
Below the two are displayed: $f(x)$ is black and the product of the two derivatives is red.



On $[-1.2, -0.9]$ (ish), $f(x)$ is decreasing but the red function is positive.

On $[-0.6, -.4]$ or so, $f(x)$ is increasing but the red function is negative.

On $[0.62, 0.8]$, $f(x)$ is decreasing but the red function is positive.

The red function is **not** the derivative of $f(x)$

$$\frac{d}{dx}\Big(f(x)g(x)\Big)$$

$$= \lim_{h\to 0} \frac{f(x+h)\;g(x+h) - f(x)\;g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\;g(x+h) - f(x)\;g(x+h) + f(x)\;g(x+h) - f(x)\;g(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)\;g(x+h) - f(x)\;g(x+h)}{h} + \frac{f(x)\;g(x+h) - f(x)\;g(}{h} \right.$$

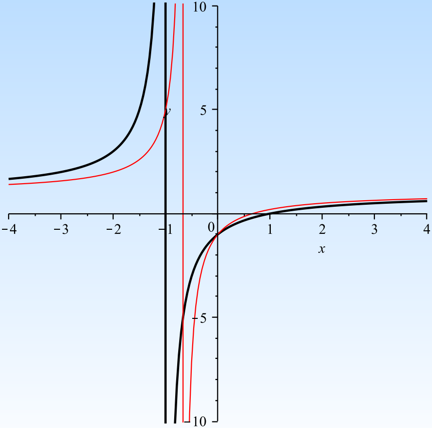
$$= \lim_{h\to 0} \left(\left[\frac{f(x+h) - f(x)}{h}\right] g(x+h) \;+\; f(x) \left[\frac{g(x+h) - g(x)}{h}\right] \right)$$

$$= f'(x)g(x) + f(x)g'(x)$$

Consider $f(x) = \dfrac{(x^3 - x^2)}{(x^3 + x^2)}$.

Is $f'(x) = \dfrac{(3x^2 - 2x)}{(3x^2 + 2x)}$?

Below the two are displayed: $f(x)$ is black and the quotient of the two derivatives is red.



$f(x)$ is always increasing, but the red function is negative for some x

On $[-1, \infty)$, $f(x)$ is increasing more and more slowly, but the red function is increasing.

The red function is **not** the derivative of $f(x)$