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Examples: Let $g(x) = e^x$, $h(x) = 3^x$, and $k(x) = 4^x$.

Remember: e is an irrational number that shows up all the time; like π it shows up enough that we give it's own name.

$$e \approx 2.71828...,$$

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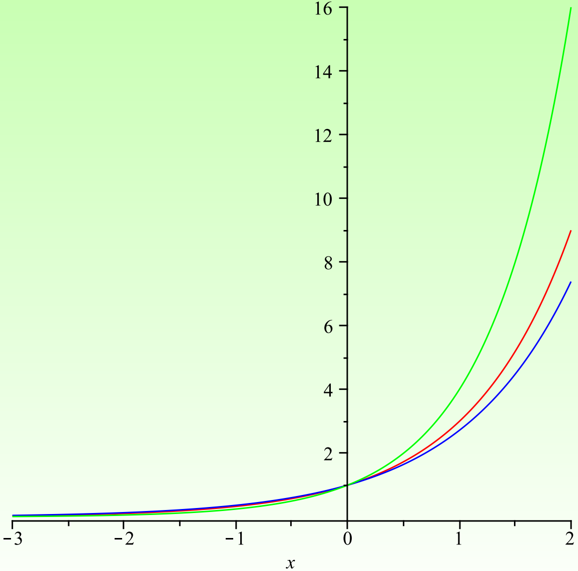
These are two fundamentally different types of functions.

Graphs of e^x, 3^x, and 4^x



Notice:

- $f(x) = b^x$ *always* passes through the points $(1, b)$ and $(0, 1)$:

$$f(1) = b^1 = b, \text{ so } (1, b) \text{ is on the graph}$$
$$f(0) = b^0 = 1, \text{ so } (0, 1) \text{ is on the graph}$$

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- $f(x) = b^x > 0$, since $b > 0$
- Domain$=(-\infty, \infty)$ and Range$=(0, \infty)$
  Remember:
  - the *domain* is the set of inputs – what x does it make sense to plug in;
  - the *range* is the set of outputs – what values of $f(x)$ will you get out, if you plug in all possible values of x?

Definition:

A **logarithm function** with base b is denoted

$$f(x) = \log_b(x)$$

and is defined by $y = \log_b(x)$ iff $b^y = x$.

Inverse Functions

- Two functions are **inverses** of each other if
  - For all $x \in$ the domain of g, $f\big(g(x)\big) = x$
    and
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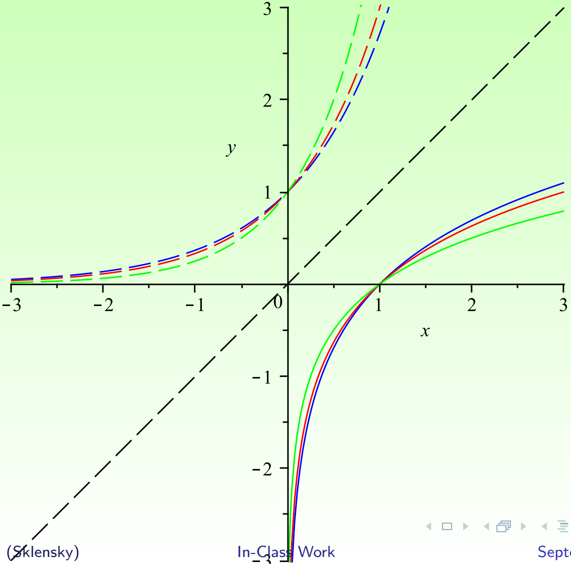
- **Notation:**
  If f and g are inverses of each other, we write $g(x) = f^{-1}(x)$.
- **Examples of Inverse Functions**
  - $f(x) = x^2$ for $x \geq 0$; $f^{-1}(x) = \sqrt{x}$
  - $f(x) = 2x + 5$, $f^{-1}(x) = \frac{x-5}{2}$

More on Inverse Functions:

- If a point (a, b) is on the graph of $f(x)$, then that means that $f(a) = b$.
- In turn, that must mean that $f^{-1}(b) = f^{-1}(f(a)) = a$, so the point (b, a) is on the graph of $f^{-1}(x)$.
- This ends up meaning that the graph of f^{-1} is the same as the graph of f reflected across the line $y = x$.

Graphs of $\ln(x)$, $\log_3(x)$, and $\log_4(x)$, along with those of e^x, 3^x, and 4^x



Notice:

- $f(x) = \log_b(x)$ *always* passes through the points $(b, 1)$ and $(1, 0)$:

$$\begin{aligned} \Box = \log_b(b) \Leftrightarrow b^{\Box} = b \Leftrightarrow \Box = 1 \\ \text{Thus } (b, 1) \text{ is on the graph} \\ \Box = \log_b(1) \Leftrightarrow b^{\Box} = 1 \Leftrightarrow \Box = 0 \\ \text{Thus } (1, 0) \text{ is on the graph} \end{aligned}$$

Notice:

- $f(x) = \log_b(x)$ *always* passes through the points $(b, 1)$ and $(1, 0)$:

$$\square = \log_b(b) \Leftrightarrow b^{\square} = b \Leftrightarrow \square = 1$$
$$\text{Thus } (b,1) \text{ is on the graph}$$
$$\square = \log_b(1) \Leftrightarrow b^{\square} = 1 \Leftrightarrow \square = 0$$
$$\text{Thus } (1,0) \text{ is on the graph}$$

- Logarithms are not defined at $x = 0$:

$$\square = \log_b(0) \Leftrightarrow b^{\square} = 0$$
$$\text{Thus 0 is not in the domain of any logarithmic function}$$

Also notice:

- Logarithms are not defined at negative numbers:

$$\square = \log_b(\text{negative number} \Leftrightarrow b^{\square} = \text{ negative number}$$

Since b is positive, no matter what power you raise it to, $b^{\square}$ will always be positive.