Recall:

Recall: We use the notation $\int_a^b f(x)\,dx$ to represent the *signed area* between $f(x)$ and the x-axis from $x = a$ to $x = b$.

We read this as *the integral of $f(x)$ from a to b.*

By *signed area*, we mean that **if** we are going from left to right (that is, if $a \leq b$), then regions above the x-axis have positive area, while regions below the x-axis have negative area.

Although we will see later that dx has some deeper historical meaning, for now the dx notation at the end of the expression serves two purposes: (1) it tells us what our variable is, in case that is not otherwise clear (just as the $\frac{d}{dx}$ notation does for differentiation), and (2) it signals that we have reached the end of the expression being integrated.

Goal:

How do you measure the area of a region in the plane?

Some shapes are easy, like squares, rectangles, triangles, circles.

What about regions bounded by a graph $y = f(x)$, the x-axis, and the lines $x = a$, $x = b$?

Or even regions bounded above *and* below by functions?

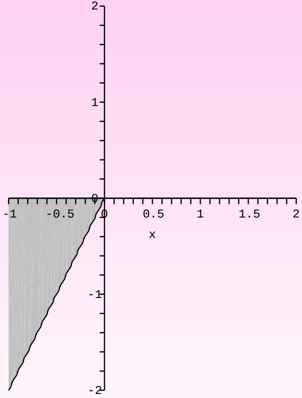
In Class Work

Using only what we know so far – that the integral is the *signed* area between the graph and the x-axis – evaluate the following integrals by graphing the function over the designated region.

1. $\displaystyle\int_0^4 2x\, dx$
2. $\displaystyle\int_{-1}^0 2x\, dx$
3. $\displaystyle\int_{-1}^4 2x\, dx$
4. $\displaystyle\int_{-1}^1 x^3\, dx$
5. $\displaystyle\int_0^\pi \cos(x)\, dx$
6. $\displaystyle\int_2^0 x+2\, dx$

Solutions

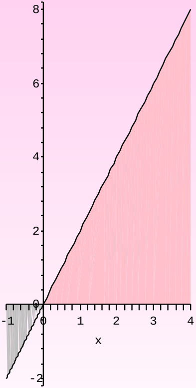
2. $\int_{-1}^{0} 2x \, dx$



- All of the area is below the x-axis, so the integral will be negative.
- This is just a triangle with base 1 and height 2.
- $\int_{-1}^{0} 2x \, dx = -\frac{1}{2}(1)(2) = -1.$

Solutions

3. $\int_{-1}^{4} 2x\,dx$



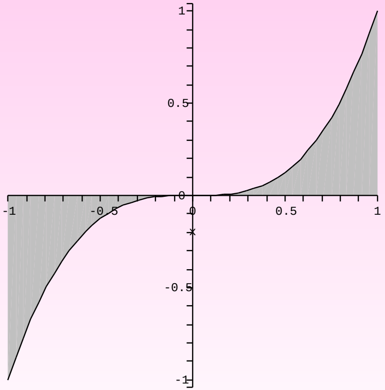
- The region from -1 to 0 is negative, and the region from 0 to 4 is positive.
- $\int_{-1}^{4} 2x\,dx = \int_{-1}^{0} 2x\,dx + \int_{0}^{4} 2x\,dx$

$$= -1 + 16$$

$$= 15.$$

Solutions

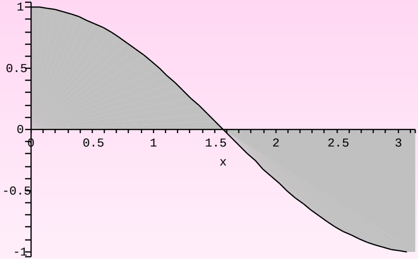
4. $\int_{-1}^{1} x^3 \, dx$



- Half of this graph is above the x-axis while the other half is below.
- The two sides are symmetric across the origin – so the negative part will cancel out the positive part.
- $\int_{-1}^{1} x^3 \, dx = 0$

Solutions

5. $\displaystyle\int_0^{\pi} \cos(x)\, dx$



As with the previous problem, the signed area of the region above the x-axis will cancel out with signed area of the region below the x-axis, so

$$\int_0^{\pi} \cos(x)\, dx = 0.$$

Solutions

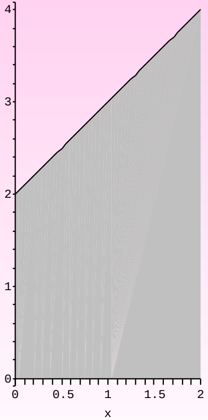
6. $\int_2^0 x + 2 \, dx$

   - The interval goes backwards: from 2 to 0.
   - With regular area, this doesn't matter.
   - Since integrals are signed area, and since they are defined as going *from* $x = a$ *to* $x = b$, direction matters.
   - Thus when moving from right to left rather than from left to right, we expect the signed area to be different.
   - The absolute value of it (the area) can't be depend on direction, so the only thing that *can* be different is the sign.

$$\boxed{\int_2^0 x + 2 \, dx = -\int_0^2 x + 2 \, dx}$$

Solutions

6. $\int_2^0 x+2\,dx = -\int_0^2 x+2\,dx$



- This region is made up of a triangle sitting on a rectangle.
- The rectangle has $b=2$ and $h=2$ (and so is a square), with area 4.
- The triangle has $b=2$ and $h=2$, so area is $\frac{1}{2}(4)=2$.
- Thus $\int_0^2 x+2\,dx = 2+4=6$
- And hence $\int_2^0 x+2\,dx = -6$.