Recall:

$$\int_0^x 2\ dt = 2x$$

$$\int_0^x t\ dt = \frac{1}{2}x^2$$

$$\int_0^x 2t\ dt = x^2$$

In every example we've looked at so far, the area function is an antiderivative of the original function. Coincidence?

Recall:

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$$\int_0^x 2t\,dt = x^2$$

$$\int_0^x t+2\,dt = \frac{1}{2}x^2 + 2x$$

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$$\int_0^x t+2\,dt = \frac{1}{2}x^2+2x$$

$$\int_0^x 7t+4\,dt = \frac{7}{2}x^2+4x$$

In every example we've looked at so far, the area function is an antiderivative of the original function. Coincidence?

In Class Work

1. $\int_0^{\pi/2} \cos(x)\, dx$

2. $\int_1^4 x^3 - 2x\, dx$

3. $\int_{-1}^2 e^x\, dx$

4. $\int_1^3 3x^2 \ln(x) + x^3 \left(\frac{1}{x}\right)\, dx$

Solutions

1. $\int_0^{\pi/2} \cos(x)\, dx$

   $\sin(x)$ is an antiderivative of $\cos(x)$, so from the FTC v2, we know

   $$\int_0^{\pi/2} \cos(x)\, dx = \sin(x) \text{ from } 0 \text{ to } \pi/2 \; = \sin(\pi/2) - \sin(0) = 1.$$

2. $\int_1^4 x^3 - 2x\, dx$

   $\frac{x^4}{4} - x^2$ is an antiderivative of $x^3 - 2x$, so from the FTC v2, we know

   $$\int_1^4 x^3 - 2x\, dx = \left(\frac{x^4}{4} - x^2\right) \text{ from } 1 \text{ to } 4 = \left(\frac{4^4}{4} - 16\right) - \left(\frac{1}{4} - 1\right) = 48 + \frac{3}{4}$$

Solutions

3. $\displaystyle\int_{-1}^{2} e^x \, dx$

e^x is of course an antiderivative of e^x, so from the FTC v2, we know

$$\int_{-1}^{2} e^x \, dx = e^x \text{ from } -1 \text{ to } 2 \; = e^2 - e^{-1}$$

Solutions

4. $\displaystyle\int_1^3 3x^2\ln(x) + x^3\left(\frac{1}{x}\right)\ dx$

If $f(x) = 3x^2\ln(x) + x^3\left(\frac{1}{x}\right)$, then I know from the FTC that

$$\int_1^3 3x^2\ln(x) + x^3\left(\frac{1}{x}\right)\ dx = F(3) - F(1),$$

where $F(x)$ is any antiderivative of $3x^2\ln(x) + x^3\left(\frac{1}{x}\right)$.

So ...

- **Goal 1:** find an antiderivative $F(x)$ of $3x^2\ln(x) + x^3\left(\frac{1}{x}\right)$.
  - Whatever F might be, it has to differentiate into this sum of two products.
  - The product rule produces a sum of two products, with related pairs of factors.
  - When I look more closely at $3x^2\ln(x) + x^3\left(\frac{1}{x}\right)$, I see that $3x^2$ is the derivative of x^3 and $\dfrac{1}{x}$ is the derivative of $\ln(x)$. Product rule it is!
  - Try: $F(x) = x^3\ln(x)$.
  - *Check:* $F'(x) = (x^3)'\cdot\ln(x) + x^3\cdot(\ln(x))' = 3x^2\ln(x) + x^3\ln(x)$
  - **Conclusion:** $F(x) = x^3\ln(x)$.

Solutions

4. $\displaystyle\int_1^3 3x^2\ln(x) + x^3\left(\frac{1}{x}\right)\ dx$

If $f(x) = 3x^2\ln(x) + x^3\left(\frac{1}{x}\right)$, then I know from the FTC that

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where $F(x)$ is any antiderivative of $3x^2\ln(x) + x^3\left(\frac{1}{x}\right)$.

- **Goal 1:** Find an antiderivative $F(x)$ of $3x^2\ln(x)$:

$$F(x) = x^3\ln(x)$$

- **Goal 2:** find the value of the definite integral
  Using FTC v2,

$$\begin{aligned}\int_1^3 3x^2\ln(x) + x^3\left(\frac{1}{x}\right)dx &= F(3) - F(1)\\ &= [3(3)^2\ln(3)] - [3(1)^2\ln(1)] = 27\ln(3).\end{aligned}$$