Recall:

- **Rolle's Theorem**: Let f be continuous on $[a, b]$ and differentiable on (a, b), *and* let $f(a) = f(b)$. Then there exists some number $c \in (a, b)$ such that $f'(c) = 0$.

  That is, there exists some point on the graph of f between $(a, f(a))$ and $(b, f(b))$ where the tangent line is horizontal.

- **The Mean Value Theorem**: Let f be continuous on $[a, b]$ and differentiable on (a, b). Then there exists some number $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

  That is, there exists some point on the graph of f between $(a, f(a))$ and $(b, f(b))$ where the tangent line is parallel to the line connecting $(a, f(a))$ to $(b, f(b))$.

Examples of Indeterminate Form

1. $\lim_{x\to\infty} \frac{e^x}{4x}$ Both the top and bottom blow up
2. $\lim_{x\to 0} \frac{\sin(3x)}{2x}$ Both go to 0
3. $\lim_{t\to\infty} te^{-t}$ One goes to ∞, the other to 0

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3. If one piece of a product $\to 0$ and the other piece is finite, product will $\to 0$; if one piece of a product $\to \infty$ and the other is non-zero, product $\to \pm\infty$. How fast is one term approaching 0 compared to rate at which other approaches ∞?