Recall:

- Local extrema can *only* occur where $f'(x) = 0$ or where $f'(x)$ d.n.e. (However, $f'(x)$ *can* be 0 or not exist at points that are *not* extrema.)
- $f(x)$ increases to the left of a local max and decreases to the right. $f(x)$ decreases to the left of a local min and increases to the right.
- Let $f(x)$ be a continuous function.

$$\begin{aligned} f(x) \text{ is increasing} \quad &\iff \quad f'(x) > 0 \\ f(x) \text{ is decreasing} \quad &\iff \quad f'(x) < 0 \end{aligned}$$

- $\therefore$ $f'(x)$ switches from $+$ on the left of a local max to $-$ on the right $\& f'(x)$ switches from $-$ on the left of a local min to $+$ on the right.

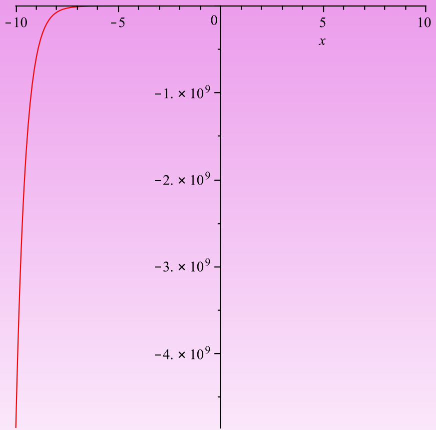
First Derivative Test:

Suppose that f is continuous and that c is a critical number for f.

- If $f'(x) > 0$ on some interval (a, c) and $f'(x) < 0$ on some interval (c, b), then f has a local maximum at $x = c$.
- If $f'(x) < 0$ on some interval (a, c) and $f'(x) > 0$ on some interval (c, b), then f has a local minimum at $x = c$.
- If $f'(x)$ has the same sign on some interval (a, c) and on some interval (c, b), then f does not have a local extremum at $x = c$.

Example: Find all local extrema of $y = xe^{-2x}$.

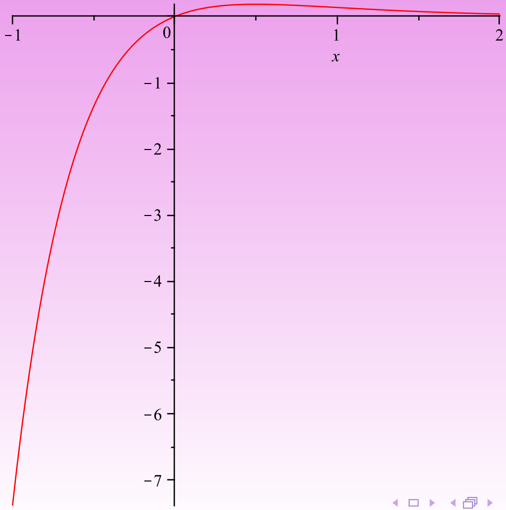
Look at the graph on the standard graphing calculator window $[-10, 10]$:



It looks as though the function is always increasing, becoming asymptotic to $y = 0$ as $x \to \infty$

Example: Find all local extrema of $y = xe^{-2x}$.

Now that we know it has a local max at $x = 1/2$, we know roughly where to zoom in- try $x \in [-1, 2]$



In Class Work

1. Find (by hand) the intervals where the function is increasing and decreasing. Use this information (and a few key points) to sketch a graph. If you have access to graphing technology, then verify your results.
   (a) $y = x^3 - 3x + 2$
   (b) $y = (x + 1)^{2/3}$
2. For the function $f(x) = (x + 1)^2 e^x$, find (by hand) all critical numbers and then use the First Derivative Test to classify each as the location of a local maximum, local minimum, or neither.

Solutions:

1. (a) $y = x^3 - 3x + 2$

- **Locate Critical Numbers**

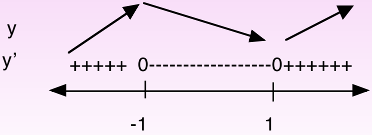
$$y' = 3x^2 - 3 \Rightarrow y' = 3(x^2 - 1) = 3(x-1)(x+1).$$

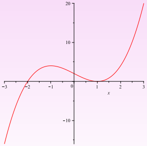
  - y' exists everywhere
  - $y' = 0$ at $x = -1$, $x = 1$.

  Thus $x = -1$ and $x = 1$ are the only two critical numbers.

- **Find where f is increasing, decreasing:**

$$f'(-2) = 3(-)(-) > 0 \quad f'(0) = 3(-)(+) < 0 \quad f'(2) = 3(+)(+) > 0$$





Solutions:

1. (b) $y = (x+1)^{2/3}$

- **Locate Critical Numbers:**

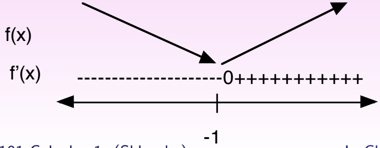
$$y' = \frac{2}{3}(x+1)^{-1/3} = \frac{2}{3(x+1)^{1/3}}.$$

  - y' does not exist at $x = -1$ (but y does)
  - $y' \neq 0$

  Thus $x = -1$ is the only critical number.

- **Find where f is increasing and decreasing:**

$$f'(-2) = \frac{2}{(+)(-)} < 0 \qquad f'(0) = \frac{2}{(+)(+)} > 0$$



2. $y = (x+1)^2 e^x$

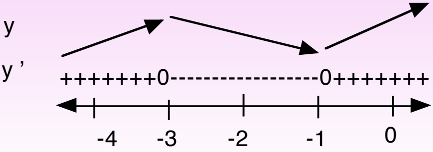
- **Locate Critical Numbers:**

$$\begin{aligned} y' &= (x+1)^2 e^x + 2(x+1)e^x \\ &= (x+1)e^x\big((x+1)+2\big) \\ &= (x+1)(x+3)e^x \end{aligned}$$

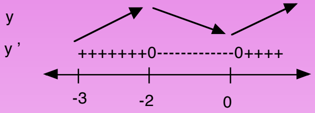
  - y' exists everywhere
  - $y' = 0$ when $x = -1$ and when $x = -3$.

  Critical numbers: $x = -1$ and $x = -3$.

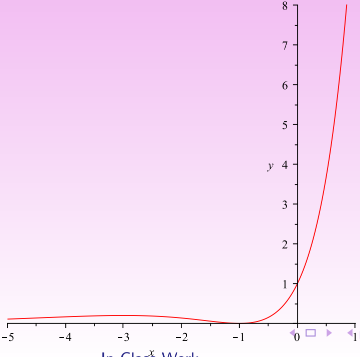
- **Find where f is increasing and decreasing:**
  - $f'(x) = (x+1)(x+3)e^x$
  - $f'(-4) = (-)(-)(+) > 0$, $f'(-2) = (-)(+)(+) < 0$,
    $f'(0) = (+)(+)(+) > 0$



Solutions:

- 

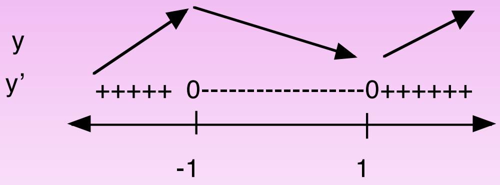
- **Conclude:**
  y has a local maximum at $x = -3$ and a local minimum at $x = -1$
- **Verify:**



Recall from Earlier Today:

Example: $f(x) = x^3 - 3x + 2$

By finding that $f'(x) = 3x^2 - 3$, you found that the critical numbers are $x = -1$ and $x = 1$, and that



But how is it curved? For that, we need concavity.

Example, continued