## Types of Functions We Can't Yet Differentiate

- $f(x)=\left(x^{6}-14 x^{5}+27 x^{-3}-13\right)\left(101 x^{-1}+14 x^{6}+13-42 \sqrt{x}\right)$
- $g(x)=\frac{x^{7}-\sqrt{x}}{14 x^{2}+12}$
- $h(x)=\left(x^{2}+1\right)^{25}$
- $j(x)=\cos \left(x^{2}\right)$
- $k(x)=\sin \left(e^{14 x}\right)$
- $m(x)=\ln (\sqrt{x}-14)$


## Comparing graphs:

If $h(x)=\left(x^{2}+1\right)^{25}$, before reading Section 2.5, many students might guess that $h^{\prime}(x)=25(2 x)^{24}$. Is it?


## For Reading Question \#1

The textbook calculated the following derivatives by expanding $\left(x^{2}+1\right)^{2}$ and $\left(x^{2}+1\right)^{3}$, differentiating the result, and then factoring those results back again:

$$
\begin{aligned}
\frac{d}{d x}\left[\left(x^{2}+1\right)^{2}\right] & =2\left(x^{2}+1\right)^{1}(2 x) \\
\frac{d}{d x}\left[\left(x^{2}+1\right)^{3}\right] & =3\left(x^{2}+1\right)^{2}(2 x)
\end{aligned}
$$

1.(a) Why would you speculate that the derivative of $\left(x^{2}+1\right)^{4}$ is $4\left(x^{2}+1\right)^{3}(2 x)$ ?

If $h(x)=\left(x^{2}+1\right)^{25}$, we found that $h^{\prime}(x)=25\left(x^{2}+1\right)^{24}(2 x)$. Do the graphs support this?


## In Class Work

Find the derivatives of the following functions:

1. $v(x)=\left(\frac{8}{x^{2}}+5 \sqrt{x}\right)\left(5 x^{-4}-\frac{7}{\sqrt[5]{x}}\right)$
2. $w(x)=\frac{x^{4}-3 \sqrt{x}}{5 x^{2}+7}$
3. $h(x)=\left(3 x^{4}+2 x\right)^{12}$
4. $j(x)=\sqrt{7-2 x^{4}}+\frac{8}{x}$
5. $k(x)=x^{5} \sqrt{x^{3}+2}$
6. $m(x)=\left(\sqrt{7 x^{6}+14 x}+2 \sqrt{x}\right)^{-2}$

## Solutions

$$
\begin{aligned}
& \text { 1. } v(x)=\left(\frac{8}{x^{2}}+5 \sqrt{x}\right)\left(5 x^{-4}-\frac{7}{\sqrt[5]{x}}\right) \\
& f(x)=\frac{8}{x^{2}}+5 x^{1 / 2} \quad g(x)=5 x^{-4}-7 x^{-1 / 5} \\
& v^{\prime}(x)=f(x) g^{\prime}(x)+f^{\prime}(x) g(x) \\
& =\left(\frac{8}{x^{2}}+5 x^{1 / 2}\right)\left(-20 x^{-5}+\frac{7}{5} x^{-6 / 5}\right) \\
& +\left(16 x+\frac{5}{2} x^{-1 / 2}\right)\left(5 x^{-4}-7 x^{-1 / 5}\right)
\end{aligned}
$$

## Solutions

$$
\begin{aligned}
& \text { 1. } \begin{aligned}
& w(x)=\frac{x^{4}-3 \sqrt{x}}{5 x^{2}+7} \\
& f(x)=x^{4}-3 x^{1 / 2} \quad g(x)=5 x^{2}+7 \\
& w^{\prime}(x)= \frac{g f^{\prime}-f g^{\prime}}{g^{2}} \\
&= \frac{\left(5 x^{2}+7\right)\left(4 x^{3}-\frac{3}{2} x^{-1 / 2}\right)-\left(x^{4}-3 x^{1 / 2}\right)(10 x)}{\left(5 x^{2}+7\right)^{2}}
\end{aligned}
\end{aligned}
$$

## Solutions

3. $h(x)=\left(3 x^{4}+2 x\right)^{12}$

This is a composition. inner: $u(x)=3 x^{4}+2 x$ outer: $f(x)=x^{12}$

$$
\begin{aligned}
h^{\prime}(x) & =f^{\prime}(u) u^{\prime} \\
& =12 u^{11} u^{\prime}(x) \\
& =12\left(3 x^{4}+2 x\right)^{11}\left(12 x^{3}+2\right)
\end{aligned}
$$

## Solutions

4. $j(x)=\sqrt{7-2 x^{4}}+\frac{8}{x}$

Rewrite: $j(x)=\left(7-2 x^{4}\right)^{1 / 2}+8 x^{-1}$
This is a sum.

- $\left(7-2 x^{4}\right)^{1 / 2}$ is a composition, with $f(u)=u^{1 / 2}$ and $u(x)=7-2 x^{4}$.
- $8 x^{-1}$ can be differentiated just with the power rule.

$$
j^{\prime}(x)=\frac{1}{2}\left(7-2 x^{4}\right)^{-1 / 2}\left(-8 x^{3}\right)-8 x^{-2}=-\frac{4 x^{3}}{\sqrt{7-2 x^{4}}}-\frac{8}{x^{2}} .
$$

## Solutions

5. $k(x)=x^{5} \sqrt{x^{3}+2}$

This is a product of $x^{5}$ and $\sqrt{x^{3}+2}$.

- $x^{5}$ requires only the power rule to differentiate it.
- $\sqrt{x^{3}+2}$ is a composition, with $f(u)=\sqrt{u}=u^{1 / 2}$ and $u=x^{3}+2$.

$$
\begin{aligned}
k^{\prime}(x) & =x^{5} \frac{d}{d x}\left(\sqrt{x^{3}+2}\right)+\left(5 x^{4}\right) \sqrt{x^{3}+2} \\
& =x^{5}\left(\frac{1}{2}\left(x^{3}+2\right)^{-1 / 2}\left(3 x^{2}\right)\right)+5 x^{4} \sqrt{x^{3}+2}
\end{aligned}
$$

## Solutions

6. $m(x)=\left(\sqrt{7 x^{6}+14 x}+2 \sqrt{x}\right)^{-2}$

Composition. Inner: $u(x)=\sqrt{7 x^{6}+14 x}+2 \sqrt{x}$; outer: $f(u)=u^{-2}$

$$
\begin{aligned}
m^{\prime}(x) & =-2 u^{-3} u^{\prime} \\
& =-2\left(\sqrt{7 x^{6}+14 x}+2 \sqrt{x}\right)^{-3} \frac{d}{d x}\left(\sqrt{7 x^{6}+14 x}+2 \sqrt{x}\right)
\end{aligned}
$$

$\sqrt{7 x^{6}+14 x}+2 \sqrt{x}=$ sum of comp. and straightforward fn.
For $\sqrt{7 x^{6}+14 x}$, inner: $u(x)=7 x^{6}+14 x$ outer: $f(u)=\sqrt{u}=u^{1 / 2}$

$$
\begin{aligned}
& \qquad \begin{aligned}
m^{\prime}(x)= & -2\left(\sqrt{7 x^{6}+14 x}+2 \sqrt{x}\right)^{-3} \cdot\left(f^{\prime}(u) u^{\prime}+2\left(\frac{1}{2}\right) x^{-1 / 2}\right) \\
= & -2\left(\sqrt{7 x^{6}+14 x}+2 \sqrt{x}\right)^{-3} \\
& \cdot\left(\frac{1}{2}\left(7 x^{6}+\underset{\text { In-Class Work }}{14 x}\right)^{-1 / 2}\left(42 x^{5}+14\right)+\underset{\text { Octoerer } 4,2010}{2\left(\frac{1}{2}\right) x^{-1 / 2}}\right)
\end{aligned}
\end{aligned}
$$

