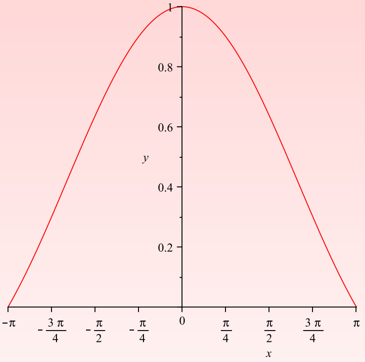
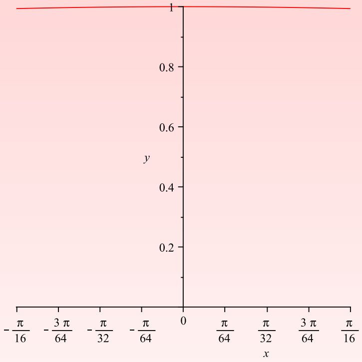
Types of Functions We Can't Yet Differentiate

- $f(x) = (x^6 - 14x^5 + 27x^{-3} - 13)(101x^{-1} + 14x^6 + 13 - 42\sqrt{x})$
- $g(x) = \dfrac{x^7 - \sqrt{x}}{14x^2 + 12}$
- $h(x) = \left(x^2 + 1\right)^{25}$
- $j(x) = \cos(x^2)$
- $k(x) = \sin(e^{14x})$
- $m(x) = \ln(\sqrt{x} - 14)$

Graphs of $\dfrac{\sin x}{x}$

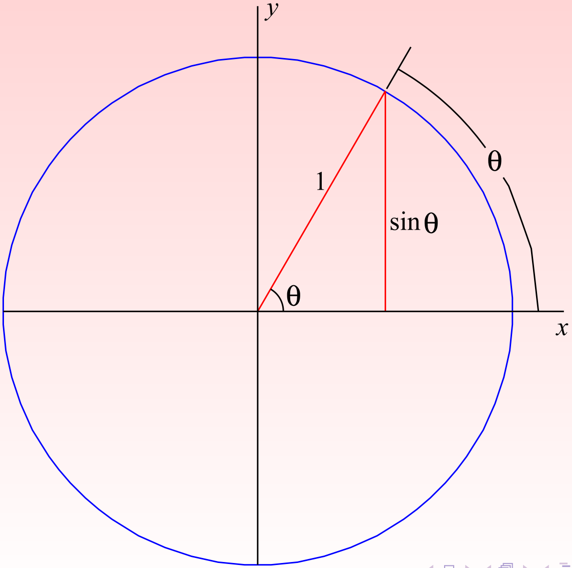


Graph of $\frac{\sin(x)}{x}$ on $[-\pi, \pi]$



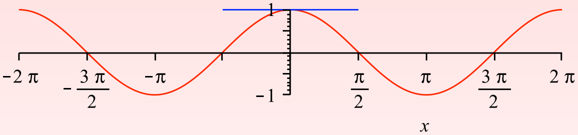
Zoom in
graph of $\frac{\sin(x)}{x}$ on $[-\frac{\pi}{16}, \frac{\pi}{16}]$

Recall - radians



Why $\lim_{h\to 0} \frac{\cos(h) - 1}{h} = 0$:

$$\lim_{h\to 0} \frac{\cos(h) - 1}{h} = \lim_{h\to 0} \frac{\cos(0+h) - \cos(0)}{h} = \frac{d}{dx}\left(\cos(x)\right)\Big|_0 .$$



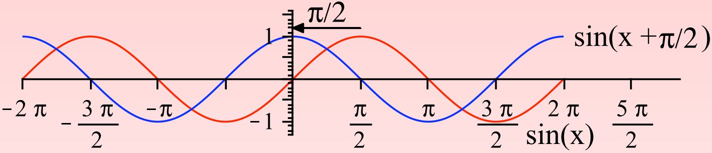
Since $\cos(x)$ clearly has a horizontal tangent line at $x = 0$, the slope at $x = 0$ is 0, so we know the derivative of $\cos(x)$ at $x = 0$ is 0, and hence this limit is 0.

Recall:

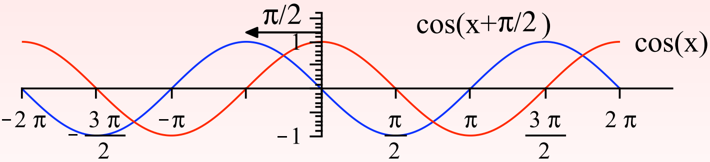
Trig identity:

$$\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h)$$

Cosine and Sine are horizontal shifts of each other



$$\sin(x + \pi/2) = \cos(x)$$



$$\cos(x + \pi/2) = -\sin(x)$$

In Class Work

1. Find the derivatives of the following:
   (a) $f(x) = \tan(x)$ (Remember, $\tan(x) = \frac{\sin(x)}{\cos(x)}$)
   (b) $g(x) = \csc(x)$ (Remember, $\csc(x) = \frac{1}{\sin(x)}$)
   (c) $h(x) = \cos(27x) + \sin(x+3)$
   (d) $j(x) = 6\sin(5x^2)$
   (e) $k(x) = \sqrt{x}\sec(x) + 7$
2. Find an antiderivative of the following; check your answers by taking the derivative.
   (a) $f(x) = \cos(x) - \sin(x)$
   (b) $h(x) = 3\sin(4) + 2\sin(3x) + x^{732} + \frac{1}{x}$

Solutions

1. Find the derivatives of the following:

   (a) $f(x) = \tan(x)$ (Remember, $\tan(x) = \frac{\sin(x)}{\cos(x)}$)

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{\cos(x)(\cos(x)) - \sin(x)(-\sin(x))}{(\cos(x))^2} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} = \left(\frac{1}{\cos(x)}\right)^2 \\
&= \sec^2(x)
\end{aligned}
$$

$$\boxed{\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)}$$

Solutions

1. Find the derivatives of the following:

   (b) $g(x) = \csc(x)$ (Remember, $\csc(x) = \frac{1}{\sin(x)}$)

$$\begin{aligned} g'(x) &= ddx\left(\big(\sin(x)\big)^{-1}\right) = -1\big(\sin(x)\big)^{-2}\big(\cos(x)\big) \\ &= -\frac{1}{\sin(x)}\frac{\cos(x)}{\sin(x)} \\ &= -\csc(x)\cot(x) \end{aligned}$$

$$\boxed{\frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cot(x)}$$

Solutions

1. Find the derivatives of the following:

   (c) $h(x) = \cos(27x) + \sin(x+3)$

   Both $\cos(27x)$ and $\sin(x+3)$ are compositions, so use the chain rule on each of them:

   $$h'(x) = -\sin(27x)(27) + \cos(x+3)(1) = -27\sin(27x) + \cos(x+3).$$

   (d) $j(x) = 6\sin(5x^2)$

   $\sin(5x^2)$ is again a composition; remember that multiplicative constants like 6 and 5 don't require the product rule.

   $$j'(x) = 6\cos(5x^2)(10x) = 60x\cos(5x^2).$$

   (e) $k(x) = \sqrt{x}\sec(x) + 7$

   Product of $\sqrt{x}$ and $\sec(x)$, so use the product rule on that part.

   $$k'(x) = \sqrt{x}\big(\sec(x)\tan(x)\big) + \left(\frac{1}{2\sqrt{x}}\right) + 0$$

Solutions

2. Find an antiderivative of the following; check your answers by taking the derivative.

   (a) $f(x) = \cos(x) - \sin(x)$

   Since
   $$\frac{d}{dx}\big(\sin(x)\big) = \cos(x) \qquad \frac{d}{dx}\big(\cos(x)\big) = -\sin(x),$$
   an *antiderivative* of $\cos(x)$ is $\sin(x)$, and an *antiderivative* of $\sin(x)$ is $-\cos(x)$.

   Therefore, one antiderivative of $f(x)$ is:
   $$F(x) = \sin(x) - \big(-\sin(x)\big) = \sin(x) + \cos(x).$$

   Check: $F'(x) = \frac{d}{dx}\big(\sin(x) + \cos(x)\big) = \cos(x) - \sin(x) = f(x)$

Solutions

2. Find an antiderivative of the following; check your answers by taking the derivative.

   (b) $h(x) = 3\sin(4) + 2\sin(3x) + x^{732} + \frac{1}{x}$

   Piece-by-piece:

   - $3\sin(4)$: Just a constant. The antiderivative of a constant k is kx, so we have $3\sin(4)x$.
   - $2\sin(3x)$: Is an antiderivative $-2\cos(3x)$? Not quite - $\dfrac{d}{dx}\big(-2\cos(3x)\big) = -\big(-2\sin(3x)(3)\big) = 6\sin(3x)$. Off by a factor of 3. Try $-\frac{2}{3}\cos(3x)$ for the antiderivative. Then $\dfrac{d}{dx}\big(-\dfrac{2}{3}\cos(3x)\big) = 2\sin(3x)$, as we want.
   - x^{732}: An antiderivative is $\dfrac{1}{733}x^{733}$
   - $\frac{1}{x}$: Rewrite as x^{-1}; an antiderivative is ????

   So an antiderivative is

$$H(x) = 3\sin(4)x - \frac{2}{3}\cos(3x) + \frac{1}{733}x^{733} + ????$$