1. Find the maximum and minimum value of $f(x) = x^3 - 3x + 5$ over the interval $[0, 2]$.

   Because f is differentiable on $[0, 2]$ (that is, because the derivative exist everywhere in $[0, 2]$), the maximum and minimum values of f can occur only at stationary points of f or at the endpoints of the interval.

   To find the stationary points, we find the derivative:

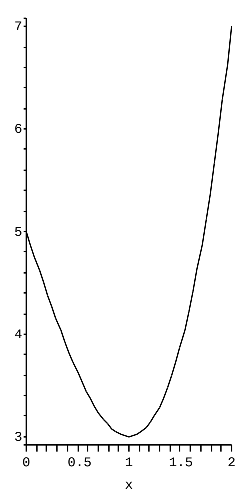
   $$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1) \qquad f'(x) = 0 \text{ at } x = 1, x = -1.$$

   Since $x = -1$ isn't in the interval $[0, 2]$, the only places f can attain its maximum and minimum value are therefore $x = 0$, $x = 1$, and $x = 2$.

x	$f(x)$
0	5
1	3
2	7

   Therefore, without even looking at the graph of $f(x)$, I know that the highest point of f between 0 and 2 occurs at $x = 2$ and the lowest occurs at $x = 1$.

   Check:



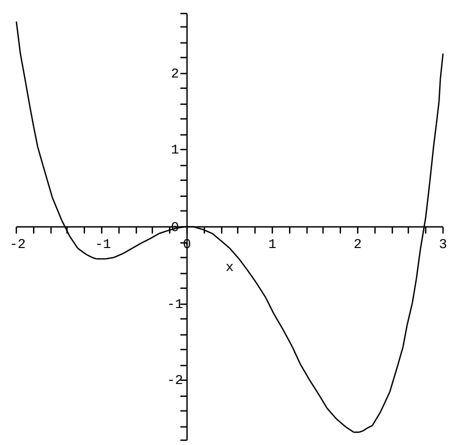
2. Find a function with stationary points at $x = -1$, $x = 0$ and $x = 2$. Check that your function behaves properly by graphing it.

   Stationary points are where the derivative is 0. So we want $f'(-1) = 0$, $f'(0) = 0$, and $f'(2) = 0$. The easiest way to achieve this is if

   $$f'(x) = (x+1)(x)(x-2) = x(x^2-2x+x-2) = x(x^2-x-2) = x^3-x^2-2x.$$

   One such function would be

   $$f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2.$$



3. For which values of k, if any, does the function $f(x) = (8x+k)/x^2$ have a local minimum at $x = 4$?

   $f(x)$ will have a local minimum at $x = 4$ if

   - $f'(4) = 0$.
   - $f''(4) > 0$ (since the function would be concave up there).

   - First, let's see what I can tell about k just from knowing $f'(4) = 0$ – that is, just from knowing that f has a stationary point at $x = 4$.

$$\begin{aligned} f(x) &= \frac{8x+k}{x^2} \\ &= \frac{8x}{x^2} + \frac{k}{x^2} \\ &= \frac{8}{x} + \frac{k}{x^2} \text{ if } x \neq 0 \\ &= 8x^{-1} + kx^{-2} \\ \Rightarrow f'(x) &= -8x^{-2} - 2kx^{-3} \\ &= -\frac{8}{x^2} - \frac{2k}{x^3} \\ \Rightarrow f'(4) &= -\frac{8}{16} - \frac{2k}{64} \\ &= -\frac{1}{2} - \frac{k}{32} \end{aligned}$$

Since $f'(4) = 0$, this means that

$$-\frac{1}{2} - \frac{k}{32} = 0 \Rightarrow \frac{k}{32} = -\frac{1}{2} \Rightarrow k = -16.$$

Conclusion: In order for f to have any sort of stationary point at $x = 4$, k must be -16. But we don't know yet whether f has a local minimum, local maximum, or inflection point at $x = 4$ when $k = -16$, so even though it feels as if we're done, we're not!

- If we let $k = -16$, is $f''(4) > 0$?
  With $k = -16$, we have that

$$\begin{aligned} f(x) &= (8x - 16)/x^2 \\ &= 8x^{-1} - 16x^{-2} \end{aligned}$$

$$\Rightarrow f'(x) = -8x^{-2} + 32x^{-3}$$
$$\Rightarrow f''(x) = -16x^{-3} - 96x^{-4}$$

$$\begin{aligned} \Rightarrow f''(4) &= \frac{16}{4^3} - \frac{96}{4^4} \\ &= \frac{1}{4} - \frac{2^5 \cdot 3}{2^8} \\ &= \frac{1}{4} - \frac{3}{8} \\ &< 0 \end{aligned}$$

  Because $f''(4)$ is negative at $x = 4$, f is concave down at $x = 4$, which means that f has a *maximum* at $x = 4$, not a *minimum* after all.

Conclusion: There is no value of k for which $x = 4$ is a local minimum of $f(x)$, although $k = -16$ does lead to $x = 4$ being a local maximum.

4. A particle moves along a straight line so that its velocity after t minutes is given by $v(t) = t^2$. How far does the particle travel between $t = 1$ and $t = 3$?

   If its velocity is given by $v(t) = t^2$, then its position must be given $p(t) = t^3/3$. Thus at $t = 1$, it was at position $p(1) = 1/3$ and at $t = 3$ it was at position $p(3) = 27/3 = 9$. At no time in that interval did it turn around, since the velocity was always positive, and so the difference in position is the same as the distance traveled.

   Thus it traveled $26/3$ units in those 2 minutes.