Let

$$f(x) = \cos(x) - \sin(x)$$
$$g(x) = 4e^x - 3\cos(x+1) - \tfrac{1}{x}$$
$$h(x) = 3\sin(4) + 2\sin(3x) - \ln(x) + x^{732}$$

1. Find the derivatives of each function.

   (a) Since we know that $\dfrac{d}{dx}(\cos(x)) = -\sin(x)$ and $\dfrac{d}{dx}(\sin(x)) = \cos(x)$, we have

   $$f'(x) = -\sin(x) - \cos(x).$$

   (b) Finding $g'(x)$ isn't quite as straightforward, so I'll go through it piece by piece.

   - The derivative of $4e^x$ is of course just $4e^x$ – we just keep the multiplicative constant in there, since if a function is stretched by a factor of 4, it grows 4 times as fast.
   - How about the derivative of $\cos(x+1)$. We've talked about this before – a horizontal shift doesn't change the shape, it just shifts it. So the slopes don't change, except for a shift by the same amount.
     If it makes you feel more comfortable, we can even refer to a Theorem. According to Theorem 4 from Section 2.2, $\dfrac{d}{dx}(s(x+a)) = s'(x+a)$. If I let $s(x) = \cos(x)$, then $\dfrac{d}{dx}(\cos(x+1)) = \dfrac{d}{dx}s(x+1) = s'(x+1)$ – in other words, I just plug $x+1$ into the derivative of $s(x) = \cos(x)$. $s'(x) = -\sin(x)$, so $\dfrac{d}{dx}(cos(x+1)) = -\sin(x+1)$.
   - Finally, we have the $\dfrac{1}{x}$ to deal with. This is not (as it's written) a power of x, nor is it an exponential function, a trig function or a log function. And yet, I seem to expect you to be able to differentiate it. That must mean it can be *rewritten*

in an equivalent form that *is* a power function, or exponential function, or log function, or trig function.
I'd assume most people are pretty comfortable that there's just no way to rewrite $\frac{1}{x}$ as an exponential, log, or trig function. So it must be a power of x. Once you have that insight, you remember! Aha! $\frac{1}{x} = x^{-1}$.
(If you don't remember ... why is this true? Well, $x \cdot \frac{1}{x} = 1$, and, because we add powers, $x \cdot x^{-1} = 1$ also. That must mean that $x^{-1} = \frac{1}{x}$.)

Putting all of that together,

$$g'(x) = 4e^x + 3\sin(x+1) + x^{-2}.$$

(c) Again, this one takes a bit of thought.

- $3\sin(4)$ is just a number, and so its derivative is simply 0.
- $2\sin(3x)$ is the same as $2\sin(x)$, except that it's compressed by a factor of 3. That means that it's changing 3 times as fast, so since $\frac{d}{dx}(2\sin(x)) = 2\cos(x)$, we must have $\frac{d}{dx}(2\sin(3x)) = 3 \cdot 2\cos(3x)$.
(Theorem 4 of Section 2.2 addresses this issue as well!)
- We know the derivative of $\ln(x)$ is $1/x$, as always.
- The derivative of x^{732} is of course $732x^{731}$.

Thus

$$h'(x) = 0 + 2 \cdot 3\cos(3x) - 1/x + 732x^{731}.$$

2. Now find the antiderivative of each function.
Check your answer by taking the derivative!

(a) I need to ask myself: what would I differentiate to get $f(x) = \cos(x) - \sin(x)$?
I know that $\frac{d}{dx}(\sin(x)) = \cos(x)$ and $\frac{d}{dx}(\cos(x)) = -\sin(x)$.
That means that I would differentiate $\sin(x)$ to get $\cos(x)$, so that gives me the first half.

What would I differentiate to get $-\sin(x)$? I'd differentiate $\cos(x)$, of course. So that gives the second part, and

$$F(x) = \sin(x) + \cos(x).$$

(b) What would I differentiate to get $4e^x - 3\cos(x+1) - \frac{1}{x}$?

Well, we've already seen that $\frac{d}{dx}(4e^x) = 4e^x$, so we know that we'd differentiate $4e^x$ to get $4e^x$.

What would we differentiate to get $3\cos(x+1)$? Based on what we found when differentiating, it makes sense that we'd differentiate $3\sin(x+1)$ to get $3\cos(x+1)$.

As for $\frac{1}{x}$, it doesn't help us when antidifferentiating this particular power of x to use the power rule, as we'd just end up with 0 in the denominator. Fortunately, we now know that the antiderivative of $\frac{1}{x}$ is $\ln(x)$. Thus

$$G(x) = 4e^x - 3\sin(x+1) - \ln(x).$$

(c) What would I differentiate to get $3\sin(4) + 2\sin(3x) - \ln(x) + x^{732}$?

Again, let's just go through it piece by piece.

- $3\sin(4)$ is just a constant m. I know that the only types of functions that have constant slope are lines – and if the slope is m, the line has the form $mx + b$ – but we don't worry about those pesky additive constants here. So the antiderivative of $3\sin(4)$ is just $3\sin(4)\cdot x$.
- When we differentiated $2\sin(3x)$, we got $3 \cdot 2\cos(3x)$. This time we're antidifferentiating. What do we get? Just go ahead and guess, and then differentiate the result!
  Let's try $2\cos(3x)$. The derivative of this is $-3 \cdot 2\sin(3x)$, when I was aiming for $2\sin(3x)$. That means I'm off by a negative sign *and* my result is 3 times too big.
  So I try $-\frac{2}{3}\cos(3x)$. The derivative of this is $-3 \cdot -\frac{2}{3}\sin(3x) = 2\sin(3x)$, which is exactly what I'm aiming for, so my second try was right!

- As for what the antiderivative of $\ln(x)$ is – you've never seen anything that when you antidifferentiate it, you get $\ln(x)$. So ... we can't do that part yet. There's still a lot more to learn, apparently!
- Skipping over that gaping hole in our result, we get to the antiderivative of x^{732}. That's just the same as it's been for a couple weeks – add 1 to the power and divide by the new power, so the antiderivative would be $x^{733}/733$.

Thus

$$H(x) = 3\sin(4)x - \frac{2}{3}\cos(3x) - ? + x^{733}/733.$$

3. Find the maximum and minimum values of $f(x)$ on the interval $[-\pi, \pi]$.

As we've seen several times now, the maximum and minimum values of $f(x)$ can occur at stationary points, or at the endpoints.

To find the stationary points, I need to take the derivative, set it equal to zero, and solve for x.

Stationary points:

$$-\sin(x) - \cos(x) = 0 \Rightarrow \sin(x) = -\cos(x) \Rightarrow x = 3\pi/4, -\pi/4.$$

Thus the only possible places the maximum and minimum value of $f(x)$ could occur are the stationary points ($x = 3\pi/4$, $x = -\pi/4$) and the endpoints ($x = -\pi$, $x = \pi$).

x	$f(x) = \cos(x) - \sin(x)$
$-\pi$	$\cos(-\pi) - \sin(-\pi) = -1$
$-\pi/4$	$\cos(-\pi/4) - \sin(-\pi/4) = \sqrt{2}/2 - (-\sqrt{2}/2) = \sqrt{2}$
$3\pi/4$	$\cos(3\pi/4) - \sin(3\pi/4) = -\sqrt{2}/2 - (\sqrt{2}/2) = -\sqrt{2}$
π	$\cos(\pi) - \sin(\pi) = -1$

Thus the minimum value of $f(x)$ on $[-\pi, \pi]$ is $-\sqrt{2}$, and it occurs at $x = 3\pi/4$; the maximum value is $\sqrt{2}$, which occurs at $x = \pi/4$.