1. Let $f(x) = \ln(x)$, $g(x) = x^2+3x$, and $h(x) = \cos(x)$. Find the following derivatives.

   (a) $(f \circ g)'(x)$
   We know from the chain rule that

$$\begin{aligned}(f \circ g)'(x) &= \frac{d}{dx} f(g(x))) \\ &= f'(g(x))g'(x) \\ &= \frac{1}{g(x)} \cdot (2x+3) \\ &= \frac{1}{x^2+3x} \cdot (2x+3)\end{aligned}$$

   (b) $(g \circ f)'(x)$
   From the chain rule, we have

$$\begin{aligned}(g \circ f)'(x) &= \frac{d}{dx}(g(f(x))) \\ &= g'(f(x))f'(x) \\ &= (2u+3) \cdot (\frac{1}{x}),\ \text{where } u = f(x) \\ &= (2\ln(x)+3) \cdot (\frac{1}{x})\end{aligned}$$

   (c) $(h \circ g)'(x)$
   The chain rule tells us

$$\begin{aligned}(h \circ g)'(x) &= \frac{d}{dx}(h(g(x))) \\ &= h'(g(x))g'(x) \\ &= \sin(g(x))(2x+3) \\ &= \sin(x^2+3x)(2x+3)\end{aligned}$$

2. Find the derivatives of the following functions.

   (a) $(3x^2+2)^{14}$

We can write this as $f(u(x))$, with $u(x) = 3x^2 + 2$ and $f(u) = u^{14}$. Therefore, the chain rule tells us that

$$\begin{aligned} \frac{d}{dx}(3x^2+2)^{14} &= f'(u)u'(x) \\ &= 14u^{13} \cdot (6x) \\ &= 14(3x^2+2)^{13}(6x) \end{aligned}$$

(b) $\ln(\sin x)$

We can write this as $f(u(x))$, with $u(x) = \sin(x)$ and $f(u) = \ln(u)$. Therefore, the chain rule tells us that

$$\begin{aligned} \frac{d}{dx}(\ln(\sin(x))) &= f'(u)u'(x) \\ &= \frac{1}{u} \cdot \cos(x) \\ &= \frac{1}{\sin(x)} \cos(x) \\ &= \cot(x) \end{aligned}$$

(c) $(\sin(3x^2))^2$

We can write this as $f(u(x))$ with $u(x) = \sin(3x^2)$ and $f(u) = u^2$. Therefore, the chain rule tells us that

$$\begin{aligned} \frac{d}{dx}((\sin(3x^2))^2) &= f'(u)u'(x) \\ &= 2u \cdot u'(x) \text{ but } u(x) \text{ is itself a composition.} \end{aligned}$$

I can write $u(x) = \sin(3x^2)$ as $g(v(x))$ with $v(x) = 3x^2$ and $g(v) = \sin(v)$. The chain rule tells us that

$$\begin{aligned} \frac{d}{dx}((\sin(3x^2))^2) &= 2u \cdot g'(v)v'(x) \\ &= 2\sin(3x^2) \cdot \cos(v) \cdot 6x \\ &= 2\sin(3x^2) \cdot \cos(3x^2) \cdot 6x \end{aligned}$$

(d) $x^2 \sin(x^3)$

If I read this out loud, it's "x^2 *times* $\sin(x^3)$." I've got two different pieces that I'm multiplying, so I have to use the product rule.

Thus I have:

$$\begin{aligned}
\frac{d}{dx}(x^2\sin(x^3)) &= \frac{d}{dx}(x^2)\cdot\sin(x^3) + x^2\cdot\frac{d}{dx}(\sin(x^3)) \\
&= 2x\sin(x^3) + x^2\frac{d}{dx}(\sin(x^3)) \text{ – I'm going to have to use the chain rule} \\
&= 2x\sin(x^3) + x^2(\cos(x^3)\cdot 3x^2)
\end{aligned}$$

(e) $\sqrt{\ln(x^2+2x)}$

I can write this as $f(u(x))$, with $u(x) = \ln(x^2+2x)$ and $f(u) = \sqrt{u} = u^{1/2}$. Therefore the chain rule tells us that

$$\begin{aligned}
\frac{d}{dx}(\sqrt{\ln(x^2+2x)}) &= f'(u)u'(x) \\
&= \frac{1}{2}u^{-1/2}\cdot\frac{d}{dx}(\ln(x^2+2x)) \text{ – I'm going to have to use the chain ru} \\
&= \frac{1}{2}(\ln(x^2+2x))^{-1/2}\cdot\frac{1}{x^2+2x}\cdot(2x+2)
\end{aligned}$$