These are only the solutions to the first six problems on the worksheet – the ones that I went over in class. The remaining solutions will be posted Wednesday after class.

Find the derivatives of the following functions.
Remember that you can verify your answers by graphing.

1. $f(x) = \sqrt{x}\cos(x)$

   This is a product, with $u = \sqrt{x} = x^{1/2}$ and $v = \cos(x)$. Thus we have

$$\begin{aligned} f'(x) &= u'v + v'u \\ &= \frac{1}{2}x^{-1/2}\cos(x) + x^{1/2}(-\sin(x)) \\ &= \frac{\cos(x)}{2\sqrt{x}} - \sqrt{x}\sin(x) \end{aligned}$$

2. $f(x) = \dfrac{3\ln(x)}{e^x}$

   This is a quotient, with $u = 3\ln(x)$ and $v = e^x$. Thus we have

$$\begin{aligned} f'(x) &= \frac{vu' - uv'}{v^2} \\ &= \frac{\frac{d}{dx}(3\ln(x)) \cdot e^x - 3\ln(x) \cdot \frac{d}{dx}(e^x)}{(e^x)^2} \\ &= \frac{\frac{3}{x} \cdot e^x - 3\ln(x)e^x}{(e^x)^2} \\ &= \frac{3e^x\left(\frac{1}{x} - \ln(x)\right)}{(e^x)^2} \\ &= \frac{3\left(\frac{1}{x} - \ln(x)\right)}{e^x} \end{aligned}$$

3. $f(x) = \dfrac{4}{(x^3 + 25x)^3}$

I could either do this using the quotient rule or just using the chain rule. I'll show you both ways, so you believe that you get the same answer either way!

- **Using the chain rule:**
  I can easily rewrite $f(x)$ as

  $$f(x) = 4(x^3 + 25x)^{-3}.$$

  When I go to differentiate it, the outer function will be $g(u) = 4u^{-3}$ and the inner function will be $u(x) = x^3 + 25x$.

  $$\begin{aligned} f'(x) &= g'(u)u'(x) \\ &= -12u^{-4}(3x^2 + 25) \\ &= -12(x^3 + 25x)^{-4}(3x^2 + 25) \\ &= -\frac{12(3x^2 + 25)}{(x^3 + 25x)^4} \end{aligned}$$

- **Using the quotient rule:**
  Here $u = 4$ (a constant! It's derivative will be 0, when the time comes) and $v = (x^3 + 25x)^3$ (a composition – I'll need the chain rule at some point, and when I do, my choices of $g(u)$ and u will be almost the same as if I'd done it only with the chain rule, as above.).
  Diving right into the quotient rule, we have

  $$\begin{aligned} f'(x) &= \frac{vu' - uv'}{v^2} \\ &= \frac{(x^3 + 25x)^3 \cdot 0 - 4 \cdot 3(x^3 + 25x)^2(3x^2 + 25)}{((x^3 + 25x)^3)^2} \\ &= -\frac{4 \cdot 3(x^3 + 25x)^2(3x^2 + 25)}{(x^3 + 25x)^6} \\ &= -\frac{12(3x^2 + 25)}{(x^3 + 25x)^4} \end{aligned}$$

  I did indeed get the same thing either way. Using the quotient rule is not the easiest way to do this problem, since you end up having to use the chain rule anyway, but knowing that it works takes some pressure off you – even if you don't recognize that you could write this as just a power, you can still get the correct answer – *as long as you remember that the derivative of a constant is 0!*

4. $f(x) = \dfrac{x\ln(x) - x}{5}$

   First of all, notice that we can write this as $f(x) = \frac{1}{5}(x\ln(x) - x)$. Since $\frac{1}{5}$ is just acting as a multiplicative constant, there's no need to use the quotient rule on this problem. However, inside the parentheses there *is* a product.

$$\begin{aligned} f'(x) &= \frac{1}{5}\left(\frac{d}{dx}(x)\ln(x) + x\frac{d}{dx}(\ln(x)) - 1\right) \\ &= \frac{1}{5}(\ln(x) + x\cdot(1) - 1) \\ &= \frac{1}{5}\ln(x) \end{aligned}$$

5. $f(x) = \dfrac{\tan(x)}{3e^x}$

   Remember, we've seen that the derivative of $\tan(x)$ is $(\sec(x))^2 = \sec^2(x)$. Now that we have that result, we don't need to re-derive it every time!

   So

$$\begin{aligned} f'(x) &= \frac{(3e^x)\frac{d}{dx}(\tan(x)) - \tan(x)\frac{d}{dx}(3e^x)}{(3e^x)^2} \\ &= \frac{3e^x(\sec(x))^2 - \tan(x)(3e^x)}{(3e^x)^2} \\ &= \frac{3e^x(\sec^2(x) - \tan(x))}{(3e^x)^2} \\ &= \frac{\sec^2(x) - \tan(x)}{3e^x} \end{aligned}$$

6. $f(x) = \sec(x)$ Remember that $\sec(x) = \dfrac{1}{\cos(x)}$.

   First of all, I notice that I can rewrite $f(x)$ some more, to avoid the quotient rule.

   $$f(x) = (\cos(x))^{-1}.$$

   Now I've got one function, $u(x) = \cos(x)$, inside another function $g(u) = u^{-1}$. Using the chain rule, I find

   $$\begin{aligned} f'(x) &= g'(u)u'(x) \\ &= -u^{-2} \cdot (-\sin(x)) \\ &= -(\cos(x))^{-2} \cdot (-\sin(x)) \\ &= \frac{\sin(x)}{(\cos(x))^2} \\ &= \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} \\ &= \tan(x)\sec(x) \end{aligned}$$