7. $f(x) = e^x \sin(x)(x^2 + 1)$

   Okay, for this we have a triple product. Remember, we saw that we can approach these by just differentiating one part of the product at a time, and leaving the other parts untouched!

   $u(x) = e^x$, $v(x) = \sin(x)$ and $w(x) = x^2 + 1$.

$$\begin{aligned} f'(x) &= u'vw + uv'w + uvw' \\ &= e^x \sin(x)(x^2+1) + e^x \cos(x)(x^2+1) + e^x \sin(x)(2x) \\ &= e^x[(x^2+1)(\sin(x) + \cos(x) + +2x\sin(x)] \end{aligned}$$

8. $f(x) = \dfrac{x\cos(x)}{\ln(x)}$

   Not to panic because we have a combination of a product and a quotient! Just take it piece by piece. The first thing we see is the quotient, with $u(x) = x\cos(x)$ and $v(x) = \ln(x)$. So we just see where that takes us:

$$f'(x) = \frac{\ln(x) \cdot \frac{d}{dx}(x\cos(x)) - x\cos(x) \cdot \frac{d}{dx}(\ln(x))}{(\ln(x))^2}.$$

   When we look at this, we see that we have to take the derivative of $x\cos(x)$ – but we know how to do that – just the product rule, no big deal.

$$\begin{aligned} f'(x) &= \frac{\ln(x) \cdot [\frac{d}{dx}(x)\cos(x) + x \cdot \frac{d}{dx}(\cos(x))] - x\cos(x)(\frac{1}{x})}{(\ln(x))^2} \\ &= \frac{\ln(x) \cdot [\cos(x) + x \cdot (-\sin(x))] - \cos(x)}{(\ln(x))^2} \\ &= \frac{\ln(x)\cos(x) - x\ln(x)\sin(x) - \cos(x)}{(\ln(x))^2} \end{aligned}$$

9. $f(x) = e^{x^3 \sin(x)}$

I've got $e^{\text{a function}}$, so I know I'm looking at the chain rule. So I dive right in: $f = g(u(x))$, where $g(u) = e^u$ and $u(x) = x^3 \sin(x)$.

(I realize as soon as I write down what u is that I'm going to have to use the product rule at some point, but I'll cross that bridge when I come to it!)

$$\begin{aligned}
f'(x) &= g'(u)u'(x) \\
&= e^u \cdot \frac{d}{dx}(x^3 \sin(x)) \text{ Here's where I need to use the product rule!)} \\
&= e^{x^3 \sin(x)}[x^3 \cos(x) + 3x^2 \sin(x)]
\end{aligned}$$

10. $f(x) = \dfrac{(x^3 + 5x)^{100}}{\ln(x)}$

Using the quotient rule, with $u = (x^3 + 5x)^{100}$ and $v = \ln(x)$, I find

$$f'(x) = \frac{\ln(x) \cdot \frac{d}{dx}\left((x^3 + 5x)^{100}\right) - (x^3 + 5x)^{100} \cdot \frac{1}{x}}{(\ln(x))^2}$$

I'm going to have to use the chain rule on $\frac{d}{dx}\left((x^3 + 5x)^{100}\right)$

The outer function is u^{100} and the inner function is $u(x) = x^3 + 5x$.

$$\begin{aligned}
&= \frac{\ln(x) \cdot 100(x^3 + 5x)^{99}(3x^2 + 5) - (x^3 + 5x)^{100} \cdot \frac{1}{x}}{(\ln(x))^2} \\
&= \frac{(x^3 + 5x)^{99}\left(100 \ln(x)(3x^2 + 5) - \frac{x^3 + 5x}{x}\right)}{(\ln(x))^2} \\
&= \frac{(x^3 + 5x)^{99}(\ln(x)(300x^2 + 500) - x^2 - 5)}{(\ln(x))^2}
\end{aligned}$$

11. $f(x) = \sec(x^3)$ (You can now use what you found the derivative of $\sec(x)$ to be.)

This is clearly a composition, with $g(u) = \sec(u)$ and $u(x) = x^3$. What ends up making this tricky is the form the derivative of $\sec(u)$ takes. Remember, we found earlier that the derivative of $\sec(u) = \sec(u)\tan(u)$. Notice that the u must appear in both the secant *and* the tangent!

So:

$$\begin{aligned} f'(x) &= g'(u)u'(x) \\ &= \sec(u)\tan(u)(3x^2) \\ &= 3x^2\sec(x^3)\tan(x^3) \end{aligned}$$

12. $f(x) = \sqrt{\cos(x^4 - \frac{7}{x})}$

Again, this is clearly a composition. I can use the chain rule, with $g(u) = \sqrt{u}$ and $u(x) = \cos(x^4 - \frac{7}{x})$. But as soon as I write this, I realize that when it comes time to differentiate $u(x)$, I'll be using the chain rule again. I'll cross that bridge when I come to it, however.

$$\begin{aligned} f'(x) &= g'(u)u'(x) \\ &= \frac{1}{2}u^{-1/2} \cdot \frac{d}{dx}\left(\cos(x^4 - \frac{7}{x})\right) \end{aligned}$$

Remember $\sqrt{u} = u^{1/2}$, which is what gives us $\frac{1}{2}u^{-1/2}$ as its derivative.

Now I'm faced with differentiating $\cos(x^4 - \frac{7}{x})$.

In order to do that, I'll use $g(u) = \cos(u)$ and $u(x) = x^4 - \frac{7}{x} = x^4 - 7x^{-1}$

$$\begin{aligned} &= \frac{1}{2}\left(\cos(x^4 - \frac{7}{x})\right)^{-1/2} \cdot g'(u)u'(x) \\ &= \frac{1}{2}\left(\cos(x^4 - \frac{7}{x})\right)^{-1/2} \cdot -\sin(u)(4x^3 + 7x^{-2}) \\ &= -\frac{1}{2}\left(\cos(x^4 - \frac{7}{x})\right)^{-1/2} \cdot \sin(x^4 - \frac{7}{x})(4x^3 + 7x^{-2}) \end{aligned}$$

13. $f(x) = \ln\left(\frac{x^3}{e^x}\right)$

I see natural log with something that's not x in it, so I know I'm using the chain rule, with $g(u) = \ln(u)$ and $u(x) = \frac{x^3}{e^x}$. I also realize, as I'm writing down what u is, that when it comes time to differentiate $u(x)$, I'm going to have to use the chain rule.

$$
\begin{aligned}
f'(x) &= g'(u)u'(x) \\
&= \frac{1}{u}u'(x) \\
&= \frac{1}{\frac{x^3}{e^x}} \cdot \frac{e^x(3x^2) - x^3(e^x)}{(e^x)^2} \\
&= \frac{e^x}{x^3} \cdot \frac{e^x(3x^2 - x^3)}{(e^x)^2} \\
&= \frac{x^2(3-x)}{x^3} \\
&= \frac{3-x}{x} \\
&= \frac{3}{x} - 1
\end{aligned}
$$

14. $f(x) = \tan(x^4)\sec^4(x)$

    Using the product rule on this, I get that

$$
\begin{aligned}
f'(x) &= \tan(x^4) \cdot \frac{d}{dx}((\sec(x))^4) + \sec^4(x) \cdot \frac{d}{dx}(\tan(x^4)) \\
&\quad \text{I'm going to have to use the chain rule on both of the derivatives left!} \\
&= \tan(x^4) \cdot 4(\sec(x))^3(\sec(x)\tan(x)) + \sec^4(x) \cdot \sec^2(x^4)(4x^3) \\
&= 4\tan(x^4)\sec^4(x)\tan(x) + 4x^3\sec^4(x)\sec^2(x^4) \\
&= 4\sec^4(x)\left(\tan(x^4)\tan(x) + x^3\sec^2(x^4)\right)
\end{aligned}
$$

15. $f(x) = \cos\left(\dfrac{1}{(x^2-5x)^4}\right)$

    I can rewrite $f(x)$ as

$$f(x) = \cos((x^2 - 5x)^{-4}).$$

This gives me an inner function, a middle function, and an outer function, so I'm looking at doing the chain rule twice. No big deal, I'll just work from the outside in, and take it as it comes.

I'll begin with $g(u) = \cos(u)$ and $u(x) = (x^2 - 5x)^{-4}$.

$$\begin{aligned}
f'(x) &= g'(u)u'(x) \\
&= -\sin(u)u'(x) \\
&= -\sin((x^2-5x)^{-4}) \cdot \frac{d}{dx}((x^2-5x)^{-4}) \\
&\quad \text{Now, I'll use } g(u) = u^{-4} \text{ and } u(x) = x^2 - 5x. \\
&= -\sin((x^2-5x)^{-4}) \cdot -4(x^2-5x)^{-5}(2x-5) \\
&= \frac{4(2x-5)\sin\left(\dfrac{1}{(x^2-5x)^4}\right)}{(x^2-5x)^5}
\end{aligned}$$

16. $f(x) = \dfrac{\ln(x^7)}{\cos(e^x)}$

This is a quotient, but both the top and the bottom are compositions. So I'm going to begin with the quotient rule, and then move right into the chain rule, when I need it.

$$\begin{aligned}
f'(x) &= \frac{\cos(e^x) \cdot \dfrac{1}{x^7} \cdot 7x^6 - \ln(x^7) \cdot -\sin(e^x)e^x}{\cos^2(e^x)} \\
&= \frac{\dfrac{7\cos(e^x)}{x} + e^x \ln(x^7)\sin(e^x)}{\cos^2(e^x)}
\end{aligned}$$

17. $f(x) = e^{\tan(\ln(x))}$

Using $g(u) = e^u$ and $u(x) = \tan(\ln(x))$ in the chain rule, I find

$$\begin{aligned}
f'(x) &= g'(u)u'(x) \\
&= e^u \cdot \frac{d}{dx}(\tan(\ln(x))) \\
&= e^{\tan(\ln(x))} \cdot \sec^2(\ln(x)) \cdot \frac{1}{x}
\end{aligned}$$

18. $f(x) = x^{1/3}\sin(x)\sec(x)$

    Another triple product!

$$\begin{aligned} f'(x) &= x^{1/3}\sin(x)\sec(x)\tan(x) + x^{1/3}\cos(x)\sec(x) + \frac{1}{3}x^{-2/3}\sin(x)\sec(x) \\ &\qquad \text{But } \sec(x) \text{ is } \frac{1}{\cos(x)} \text{ and } \tan(x) = \frac{\sin(x)}{\cos(x)}, \text{ so ...} \\ &= x^{1/3}(\tan(x))^2 + x^{1/3} + \frac{1}{3}x^{-2/3}\tan(x) \end{aligned}$$