1. $\dfrac{x-7\tan(x)}{x^3e^x}$

$$\begin{aligned}\frac{d}{dx}\left(\frac{x-7\tan(x)}{x^3e^x}\right) &= \frac{(x^3e^x)(1-7\sec^2(x))-(x-7\tan(x))(x^3e^x+3x^2e^x)}{(x^3e^x)^2}\\ &= \frac{x^2e^x[x(1-7\sec^2(x))-(x-7\tan(x))(x+3)}{x^6e^{2x}}\\ &= \frac{x-7x\sec^2(x)-x^2-3x+7x\tan(x)+21\tan(x)}{x^4e^x}\\ &= \frac{-2x-x^2-21\tan(x)+7x(\sec^2(x)+\tan(x))}{x^4e^x}\end{aligned}$$

2. $\cos(\sqrt{x-5}\ln(x))$

$$\begin{aligned}\frac{d}{dx}\left(\cos(\sqrt{x-5}\ln(x))\right) &= -\sin(\sqrt{x-5}\ln(x))[\sqrt{x-5}\cdot\frac{1}{x}+\ln(x)\cdot\frac{1}{2}(x-5)^{-1/2}\cdot(1)]\\ &= -\sin(\ln(x)\sqrt{x-5})[\frac{\sqrt{x-5}}{x}+\frac{\ln(x)}{2\sqrt{x-5}}]\end{aligned}$$

3. $\left(\dfrac{\sec(x)-\frac{1}{x^5}}{8\sin(x)-4x^5}\right)^{1/3}$

$$\begin{aligned}\frac{d}{dx}\left(\frac{\sec(x)-\frac{1}{x^5}}{8\sin(x)-4x^5}\right)^{1/3} &= \frac{1}{3}\left(\frac{\sec(x)-\frac{1}{x^5}}{8\sin(x)-4x^5}\right)^{-2/3}\\ &\quad\cdot\left(\frac{(8\sin(x)-4x^5)(\sec(x)\tan(x)+5x^{-6})}{(8\sin(x)-4x^5)^2}\right.\\ &\quad\left.-\frac{(\sec(x)-\frac{1}{x^5})(8\cos(x)-20x^4)}{(8\sin(x)-4x^5)^2}\right)\end{aligned}$$

4. $\ln(4x^5)(32\sqrt{x}-x)^{-1}$

$$\begin{aligned}
\frac{d}{dx}\left(\ln(4x^5)(32\sqrt{x}-x)^{-1}\right) &= \ln(4x^5)\cdot(-1)(32\sqrt{x}-x)^{-2}\cdot(32(\frac{1}{2})x^{-1/2}-1) \\
&\quad +(32\sqrt{x}-x)^{-1}\cdot\frac{1}{4x^5}\cdot(20x^4) \\
&= -\frac{\ln(4x^5)\left(\frac{16}{\sqrt{x}}-1\right)}{(32\sqrt{x}-x)^2}+\frac{5}{x(32\sqrt{x}-x)}
\end{aligned}$$

5. $x^4\cos(x)\sin(x)\ln(x)$

$$\begin{aligned}
\frac{d}{dx}\left(x^4\cos(x)\sin(x)\ln(x)\right) &= x^4\cos(x)\sin(x)\left(\frac{1}{x}\right)+x^4\cos(x)\cos(x)\ln(x) \\
&\quad +x^4(-\sin(x))\sin(x)\ln(x)+4x^3\cos(x)\sin(x)\ln(x) \\
&= x^3\cos(x)\sin(x)(1+4\ln(x))+x^4\ln(x)(\cos^2(x)-\sin^2(x))
\end{aligned}$$

6. $e^{\sec(x)/x}$

$$\frac{d}{dx}\left(e^{\sec(x)/x}\right) = e^{\sec(x)/x}\cdot\left(\frac{x\sec(x)\tan(x)-\sec(x)}{x^2}\right)$$

7. $3e^{7x}\sqrt{\sin(x^2)+\cos(x^3)}$

$$\begin{aligned}
\frac{d}{dx}\left(3e^{7x}\sqrt{\sin(x^2)+\cos(x^3)}\right) &= 3e^{7x}\left(\frac{1}{2}\right)(\sin(x^2)+\cos(x^3))^{-1/2}(\cos(x^2)2x-\sin(x^3)3x^2) \\
&\quad +\sqrt{\sin(x^2)+\cos(x^3)}\cdot 3e^{7x}\cdot 7 \\
&= 3e^{7x}\left(\frac{2x\cos(x^2)-3x^2\sin(x^3)}{2\sqrt{\sin(x^2)+\cos(x^3)}}+7\sqrt{\sin(x^2)+\cos(x^3)}\right)
\end{aligned}$$

8. $(x^2\tan(x)+8)^{1/5}$

$$\frac{d}{dx}\left((x^2\tan(x)+8)^{1/5}\right) = \frac{1}{5}(x^2\tan(x)+8)^{-4/5}(2x\tan(x)+x^2\sec^2(x))$$

9. $\dfrac{\ln(\pi x^{11} - 42x)}{e^{\cos(x)}}$

$$\begin{aligned} f'(x) &= \frac{e^{\cos(x)} \cdot \dfrac{1}{\pi x^{11} - 42x} \cdot (11\pi x^{10} - 42) - \ln(\pi x^{11} - 42x) \cdot e^{\cos(x)}(-\sin(x))}{(e^{\cos(x)})^2} \\ &= \frac{11\pi x^{10} - 42}{e^{\cos(x)}(\pi x^{11} - 42x)} + \frac{\sin(x)\ln(\pi x^{11} - 42x)}{e^{\cos(x)}} \end{aligned}$$

10. $\dfrac{\cos(x^2)\cos(x^3)\cos(x^4)}{e}$

$$\begin{aligned} f'(x) &= \frac{1}{e}[-2x\sin(x^2)\cos(x^3)\cos(x^4) - 3x^2\cos(x^2)\sin(x^3)\cos(x^4) \\ &\quad -4x^3\cos(x^2)\cos(x^3)\sin(x^4)] \end{aligned}$$

11. $\dfrac{x\sin(x) - x^2}{5e^x\ln(x)}$

$$f'(x) = \frac{(5e^x\ln(x))(x\cos(x) + \sin(x) - 2x) - (x\sin(x) - x^2)(5e^x\ln(x) + 5e^x \cdot \frac{1}{x})}{(5e^x\ln(x))^2}$$

12. $\sec(x\cos(x))$

$$f'(x) = \sec(x\cos(x))\tan(x\cos(x))(\cos(x) - x\sin(x))$$

13. $\sin(\cos(\ln(e^{x^2})))$

$$f'(x) = \cos(\cos(\ln(e^{x^2}))) \cdot -\sin(\ln(e^{x^2})) \cdot \left(\frac{1}{e^{x^2}}\right) \cdot e^{x^2} \cdot 2x$$

If you happen to recognize that $\ln(e^{x^2}) = x^2$, that eliminates some of the chain rule, and in that case you end up with

$$f'(x) = \cos(\cos(x^2)) \cdot -\sin(x^2) \cdot 2x$$

While the second result is significantly shorter than the first, they are, in fact, the same.

14. $\sqrt{\dfrac{5+47x}{10-13x^3}}$

Rewriting the problem first, I get

$$f(x) = \left(\frac{5+47x}{10-13x^3}\right)^{1/2}.$$

$$f'(x) = \frac{1}{2}\left(\frac{5+47x}{10-13x^3}\right)^{-1/2} \cdot \frac{(10-13x^3)(47)-(5+47x)(-39x^2)}{(10-13x^3)^2}$$

15. $e^{(x^2+5)^3}(x^2+5)^3$

This is a product, of course. Making it more confusing, perhaps, the inside of the exponential function is the same as what the exponential function is being multiplied by. Don't worry about it, just go through it step by step.

$$\begin{aligned} f'(x) &= u'v + uv' \\ &= \left(e^{(x^2+5)^3} \cdot 3(x^2+5)^2(2x)\right) \cdot (x^2+5)^3 + \left(e^{(x^2+5)^3}\right) \cdot 3(x^2+5)^2(2x) \\ &= e^{(x^2+5)^3} \cdot (6x(x^2+5)^5 + 6x(x^2+5)^2) \\ &= 6x(x^2+5)^2 e^{(x^2+5)^3}((x^2+5)^3+1) \end{aligned}$$

16. $\dfrac{e^{x^2}}{\sqrt{x}(x^3-7x)^{10}}$

$$f'(x) = \frac{\sqrt{x}(x^3-7x)^{10} \cdot e^{x^2}(2x) - e^{x^2} \cdot \left(\frac{1}{2}x^{-1/2}(x^3-7x)^{10} + \sqrt{x} \cdot 10(x^3-7x)^9(3x^2-7)\right)}{(\sqrt{x}(x^3-7x)^{10})^2}$$

17. $\sec((x+3)\sqrt{3x^4-12})\tan(x^2+\pi)$

It is important to pay attention to parentheses! Here the product of $x+3$ and $\sqrt{3x^4-12}$ is *inside* the secant function, while the tangent function is outside of it.

So I have a product of two functions, one of which has a product inside it.

$$\begin{aligned} f'(x) &= [\sec((x+3)\sqrt{3x^4-12})\tan((x+3)\sqrt{3x^4-12}) \cdot \\ &\cdot((x+3)(1/2)(3x^4-12)^{-1/2})(12x^3) + \sqrt{3x^4-12}\cdot 1)] \cdot \tan(x^2+\pi) \\ &+ \sec((x+3)\sqrt{3x^4-12}) \cdot \sec^2(x^2+\pi)(2x) \end{aligned}$$

18. $\cos(x\cos(x\cos(x\cos(x))))$

$$\begin{aligned} f'(x) &= -\sin(x\cos(x\cos(x\cos(x)))) \cdot \\ &\cdot[\cos(x\cos(x\cos(x))) - x\sin(x\cos(x\cos(x))) \cdot \\ &[\cos(x\cos(x)) - x\sin(x\cos(x)) \cdot [\cos(x) - x\sin(x)]]] \end{aligned}$$