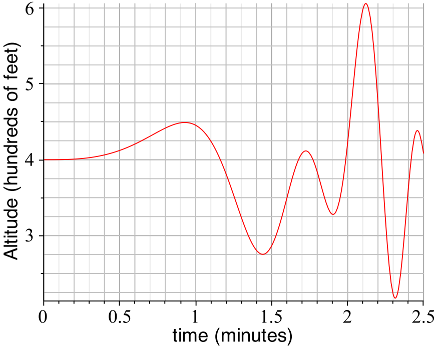
Recall: Connection between rate and slope

The graph of the altitude $A(t)$ of a hot air ballon after t minutes.



1. Slope $> 0 \Leftrightarrow$ Graph of $A(t) \uparrow \Leftrightarrow$ Altitude increasing $\Leftrightarrow$ Altitude changing at a positive rate
2. At $t = 0.9$, slope small, positive $\Leftrightarrow$ graph of $A(t)$ fairly flat $\Leftrightarrow$ balloon goes up slowly
3. At $t = 0.9$, graph concave down $\Leftrightarrow$ positive slope decreasing $\Leftrightarrow$ increasing graph getting flatter $\Leftrightarrow$ balloon going up more slowly

Recall: Estimating the rate $f(t) = e^t$ changes at $t = 1$

To summarize:

$x = 1$ to $x =?$	Average rate of change	$x =?$ to $x = 1$	Average rate of change
2	$\dfrac{e^2 - e^1}{2 - 1} \approx 4.6708$	0	$\dfrac{e^1 - e^0}{1 - 0} \approx 1.7183$
1.5	$\dfrac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$	0.5	$\dfrac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
1.1	$\dfrac{e^{1.1} - e^1}{1.1 - 1} \approx 2.8588$	0.9	$\dfrac{e^1 - e^{0.9}}{1 - 0.9} \approx 2.5868$
1.01	$\dfrac{e^{1.01} - e^1}{1.01 - 1} \approx 2.7319$	0.99	$\dfrac{e^1 - e^{0.99}}{1 - 0.99} \approx 2.7047$
1.001	$\dfrac{e^{1.001} - e^1}{1.001 - 1} \approx 2.7196$	0.999	$\dfrac{e^1 - e^{0.999}}{1 - 0.999} \approx 2.7169$
1.0001	$\dfrac{e^{1.0001} - e^1}{1.0001 - 1} \approx 2.7184$	0.9999	$\dfrac{e^1 - e^{0.9999}}{1 - 0.9999} \approx 2.7181$

Recall: Slope of Secant Line =Ave. Rate of Change

If we are looking at an object whose position is given by $f(t)$, then

$$\text{average r.o.c. from } t = c \text{ to } t = d = \frac{\text{change in position}}{\text{time}}$$
$$= \frac{f(t_{\text{final}}) - f(t_{\text{initial}})}{t_{\text{Final}} - t_{\text{initial}}}$$
$$= \boxed{\frac{f(d) - f(c)}{d - c}.}$$

If we are looking at the graph of $y = f(x)$, then

$$\text{slope of secant line from } x = c \text{ to } x = d = \frac{\text{rise}}{\text{run}}$$
$$= \frac{\Delta y}{\Delta x}$$
$$= \boxed{\frac{f(d) - f(c)}{d - c}}$$

Recall: Estimating the slope of the line tangent to $f(t) = e^t$ at $t = 1$.

$t = 1$	Slope from $(1, e^1)$	$t = ?$	Slope from (t, e^t)
2	$\frac{e^2 - e^1}{2 - 1} \approx 4.6708$	0	$\frac{e^1 - e^0}{1 - 0} \approx 1.7183$
1.5	$\frac{e^{1.5} - e^1}{1.5 - 1} \approx 3.52681$	0.5	$\frac{e^1 - e^{0.5}}{1 - 0.5} \approx 2.13912$
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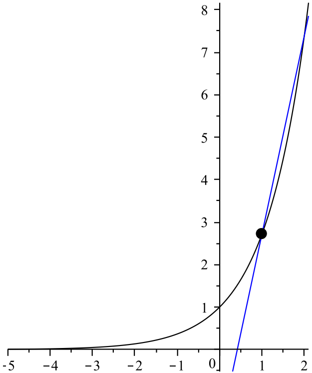
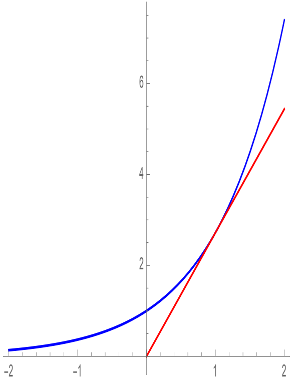
Tangent Line to $f(x) = e^x$ at $x = 1$

Secant Line from $(1, e^1)$ to $(2, e^2)$



Tangent Line *at* $(1, e^1)$



What We're Interested In:

- **Slope of a Curve (i.e, slope of the line tangent to the curve)**

- **Instantaneous Rate of Change**

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- **Slope of a Curve (i.e, slope of the line tangent to the curve)**

  The closer d is to c, the closer the slope of the secant line is to the slope of the curve itself at $\big(c, f(c)\big)$.

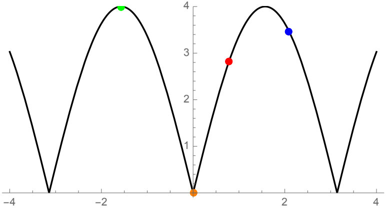
- **Instantaneous Rate of Change**

  The smaller the time interval, the closer the average rate of change is to the instantaneous rate of change at time c.

$\therefore$ **Slope of Curve at $\big(c, f(c)\big)$ = Inst. Rate of Change at time c**

Tangent Line Corresponds to Line Seen when Zoom In

Let $f(x) = \left|4\sin(x)\right|$. What happens when we zoom in on each of the four points shown?



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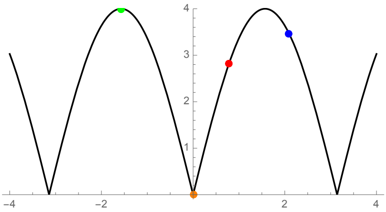

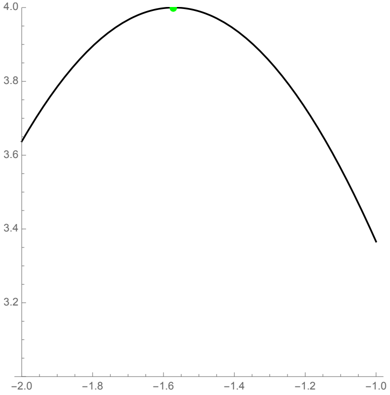

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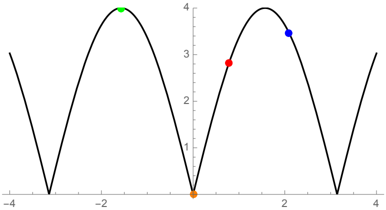

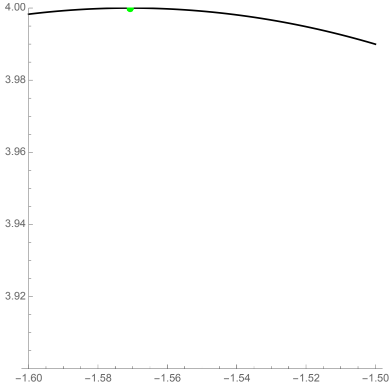

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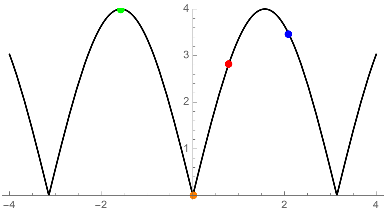

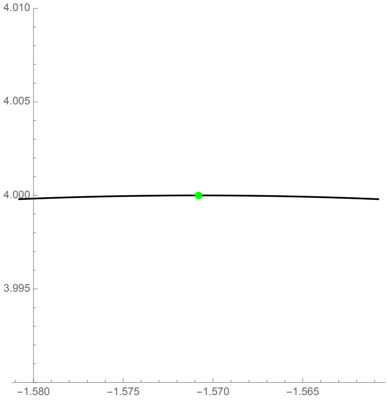

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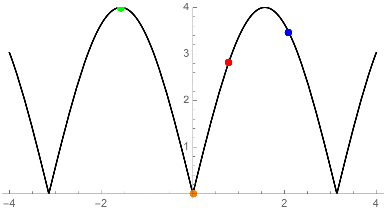

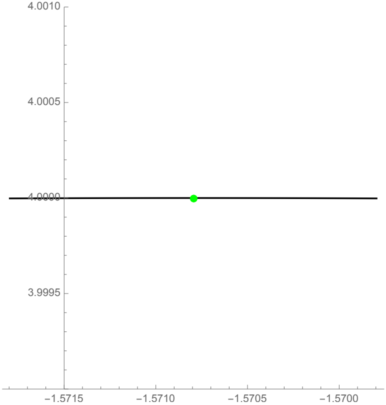

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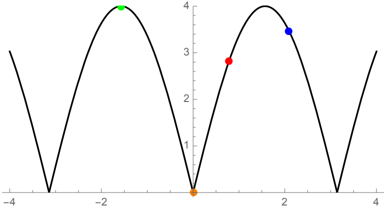

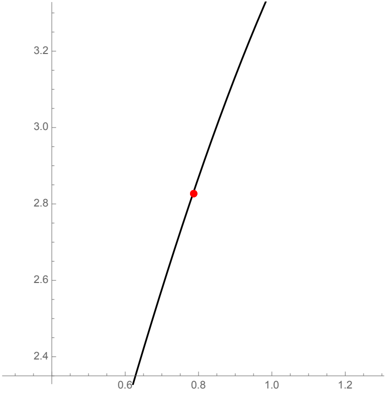

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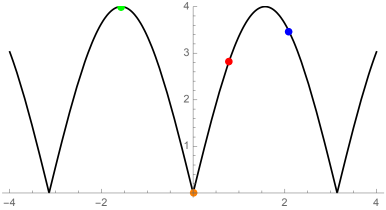

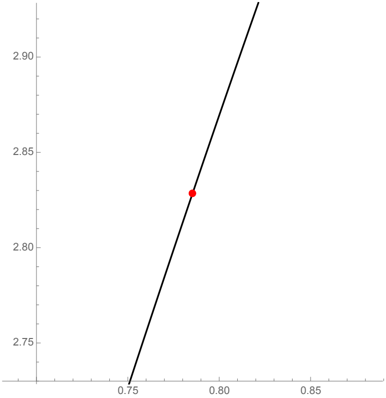

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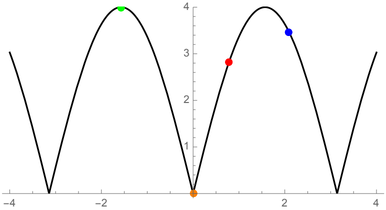

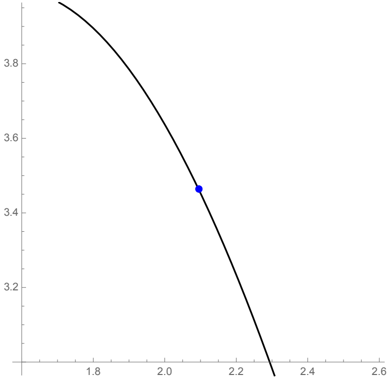

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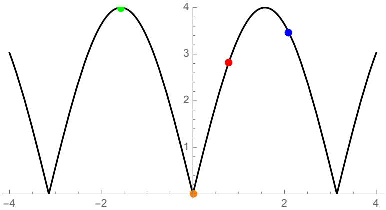

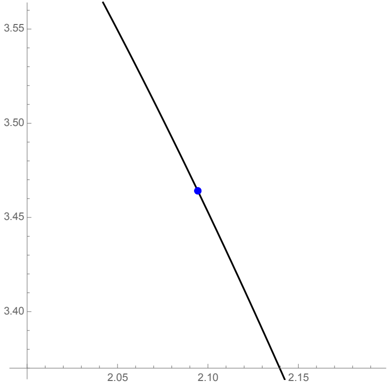

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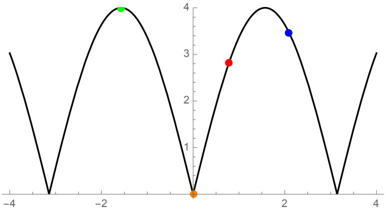

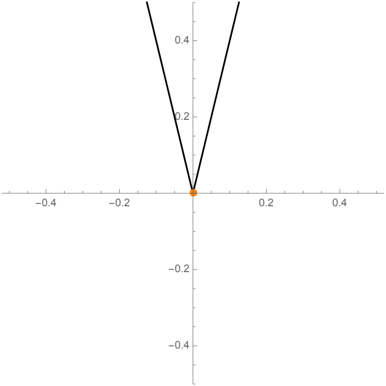

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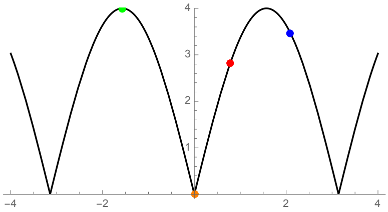

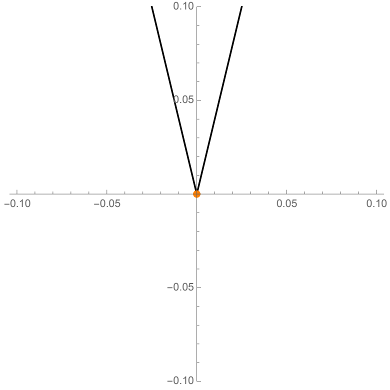

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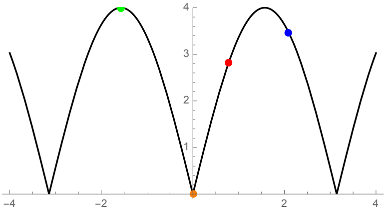
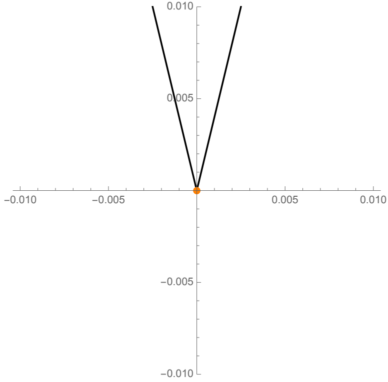
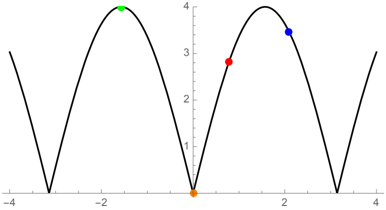
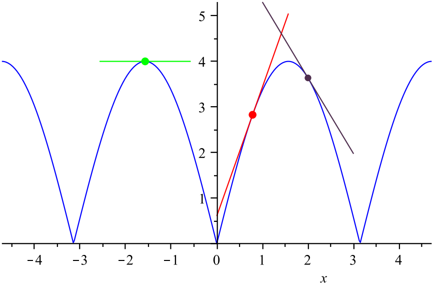


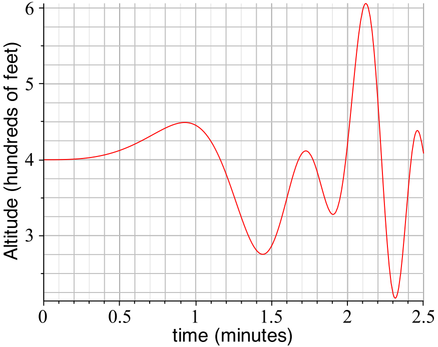



4
3
2
1
−4
−2
2
4


5
4
3
2
1
−4
−3
−2
−1
0
1
2
3
4
x

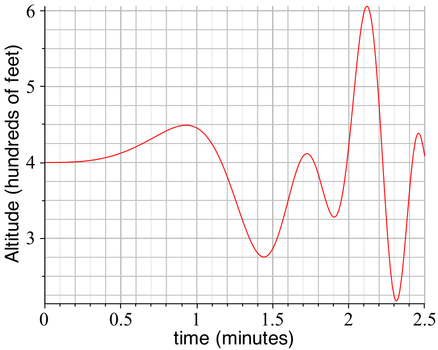
Recall: Connection between rate and slope

The graph of the altitude $A(t)$ of a hot air ballon after t minutes.



1. Slope $> 0 \Leftrightarrow$ Graph of $A(t) \uparrow \Leftrightarrow$ Altitude increasing $\Leftrightarrow$ Altitude changing at a positive rate
2. At $t = 0.9$, slope small, positive $\Leftrightarrow$ graph of $A(t)$ fairly flat $\Leftrightarrow$ balloon goes up slowly
3. At $t = 0.9$, graph concave down $\Leftrightarrow$ positive slope decreasing $\Leftrightarrow$ increasing graph getting flatter $\Leftrightarrow$ balloon going up more slowly

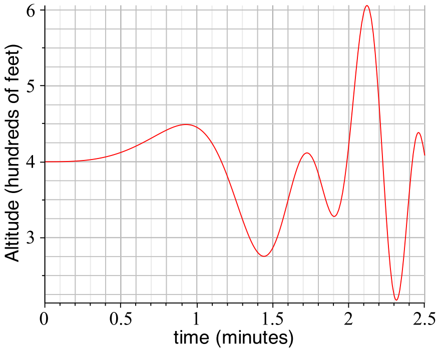
Given graph of $A(t)$, what can we say about $A'(t)$?



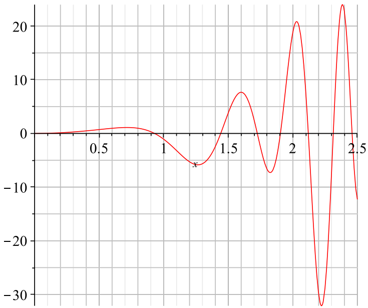
- $A(t)$ is flat at $t = 0, 0.92$, 1.42, 1.72, 1.91, 2.12, 2.32, and 2.45. At these t, balloon has upward velocity=0 & slope of $A(t)$ is 0. $A'(t) = 0$ at these points.
- On the intervals $(0, 0.92) \cup (1.42, 1.72) \cup (1.91, 2.12) \cup (2.32, 2.45)$, $A(t)$ increases, so $A(t)$ has positive slope (balloon has positive upward velocity), so the derivative $A'(t) > 0$.

Notice: $A(t)$ is steepest at inflection points (i.e. where A changes concavity). That is, the slope is locally the most negative or the most positive at these points. Thus the derivative $A'(t)$ has local minima and maxima where $A(t)$ has inflection points.

Graph of $A(t)$



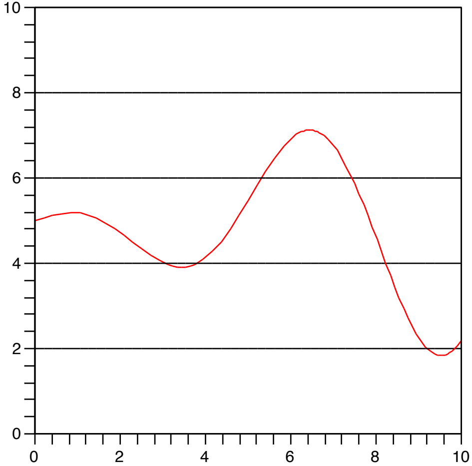
Graph of $A'(t)$



- $A(t)$ flat $\Rightarrow 0 = A'(0) = A'(0.92) = A'(1.42) = A'(1.72) = A'(1.91)$ $= A'(2.12) = A'(2.32) = A'(2.45)$
- $A(t) \uparrow \Rightarrow A'(t) > 0$ on $(0, 0.92) \cup (1.42, 1.72) \cup (1.91, 2.12) \cup (2.32, 2.45)$
- $A(t) \downarrow \Rightarrow A'(t) < 0$ elsewhere
- $A'(t) > 0$ & inflection pts $\Rightarrow A'(t)$ has local max at $t = 0.75, 1.6, 2.02, 2.38$
- $A'(t) < 0$ & inflection pts $\Rightarrow A'(t)$ has local min at $t = 1.22, 1.81, 2.21$

In Class Work

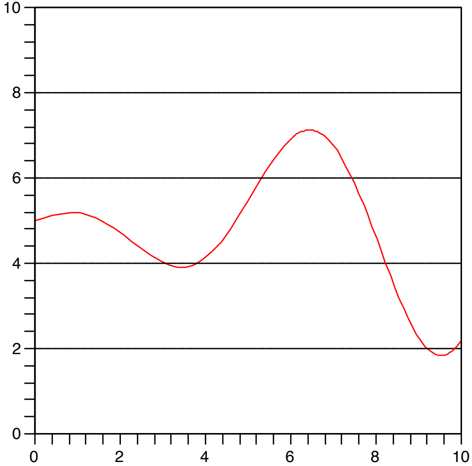
The graph gives the position $P(t)$ of a highway patrol car on the Mass Pike in miles east of Worcester, where t is minutes after 12:00 noon. The derivative $P\,'(t)$ gives the car's eastward velocity at time t.



1. At what t does the car change direction?
2. Where is $P\,'(t)$ zero? positive? negative?
3. Use this information to *sketch* a graph $P\,'(t)$.

Solutions to In Class Work

The graph gives the position $P(t)$ of a highway patrol car on the Mass Pike in miles east of Worcester, where t is minutes after 12:00 noon. The derivative $P'(t)$ gives the car's eastward velocity at time t.



1. At what t does the car change direction?

   It changes direction when the number of miles east from the pike goes from increasing to decreasing or from decreasing to increasing.
   That is, at the local maxima and local minima of $P(t)$.
   So at $t = 1$, $t = 3.6$, $t = 6.4$, and $t = 9.5$.

Solutions to In Class Work

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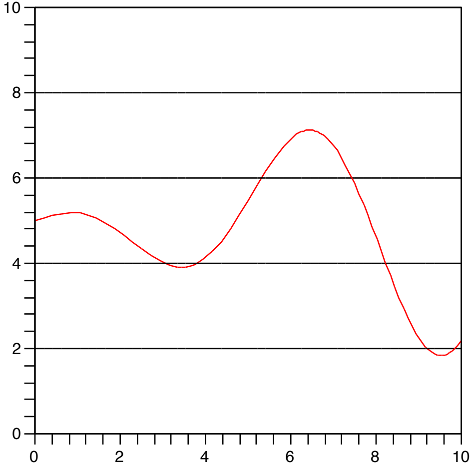

2. Where is $P'(t)$ zero? positive? negative?

   P' is zero where car changes direction, so at $x = 1$, $x = 3.6$, $x = 6.4$, $x = 9.5$.
   P' is positive where P is increasing: on $[0, 1]$, $[3.6, 6.4]$, $[9.5, 10]$
   P' is negative on $[1, 3.6]$, $[6.4, 9.5]$