Daily WW

1. Suppose that $f(x+h)-f(x) = -4hx^2 - 8hx - 5h^2x + 8h^2 + 1h^3$. Find $f'(x)$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{-4hx^2 - 8hx - 5h^2x + 8h^2 + h^3}{h}$$
$$= \lim_{h\to 0}\left(-4x^2 - 8x - 5hx + 8h + h^2\right) = -4x^2 - 8x$$

4. Let $f(x) = \dfrac{1}{x-3}$. Use the limit definition of f' ...

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{x+h-3}-\frac{1}{x-3}}{h}$$
$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{x+h-3}-\frac{1}{x-3}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{(x-3)-(x+h-3)}{(x+h-3)(x-3)}\right)$$
$$= \lim_{h\to 0}\frac{1}{h}\frac{-h}{(x+h-3)(x-3)} = \lim_{h\to 0}\frac{-1}{(x+h-3)(x-3)} = -\frac{1}{(x-3)^2}$$

Recall:

The derivative function $f'(x)$ gives the slope of the line tangent to the graph of $y = f(x)$ at every point x where such a tangent line exists.

Equivalently, the derivative $f'(x)$ gives the instantaneous rate that $f(x)$ is changing (with respect to x) at every point x.

The derivative function can be calculated using the definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Derivatives So Far:

- If $f(x) = k$, $f'(x) = 0$.
  If $f(x) = 1 = x^0$, $f'(x) = 0$ If $g(x) = k = k \cdot 1$, $g'(x) = 0 = k \cdot 0$
- If $f(x) = mx + b$, $f'(x) = m$.
  If $f(x) = x = x^1$, $f'(x) = 1 = 1x^0$ If $g(x) = kx = k \cdot x$,
  $g'(x) = k = k \cdot 1$
  If $h(x) = kx + c$, $h'(x) = k$
- If $f(x) = x^2$, $f'(x) = 2x$
  If $h(x) = x^2 + k$, $h'(x) = 2x$ If $j(x) = x^2 - 3x$, $j'(x) = 2x - 3$
- If $f(x) = \dfrac{1}{x} = x^{-1}$, $f'(x) = -\dfrac{1}{x^2} = -x^{-2}$
  If $m(x) = \dfrac{1}{x+c}$, $m'(x) = -\dfrac{1}{(x+c)^2}$
- If $f(x) = \sqrt{x} = x^{1/2}$, $f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$
  If $m(x) = c + k\sqrt{x}$, $m'(x) = \frac{k}{2}x^{-1/2}$

Derivatives So Far:

- If $f(x) = 1 = x^0$, $f'(x) = 0$
- If $f(x) = x = x^1$, $f'(x) = 1 = 1x^0$
- If $f(x) = x^2$, $f'(x) = 2x$
- If $f(x) = \dfrac{1}{x} = x^{-1}$, $f'(x) = -\dfrac{1}{x^2} = -x^{-2}$
- If $f(x) = \sqrt{x} = x^{1/2}$, $f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$

Remember: Pascal's Triangle

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

$\vdots \qquad \vdots \qquad \vdots$

Remember: Pascal's Triangle

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

$$\vdots$$

$$1$$
$$x + h$$
$$x^2 + 2xh + h^2$$
$$x^3 + 3x^2h + 3xh^2 + h^3$$
$$x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$
$$x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$
$$\vdots$$

Remember: Pascal's Triangle

1	1	$(x+h)^0$
1 1	$x+h$	$(x+h)^1$
1 2 1	$x^2+2xh+h^2$	$(x+h)^2$
1 3 3 1	$x^3+3x^2h+3xh^2+h^3$	$(x+h)^3$
1 4 6 4 1	$x^4+4x^3h+6x^2h^2+4xh^3+h^4$	$(x+h)^4$
1 5 10 10 5 1		$(x+h)^5$
$\vdots$	$\vdots$	$\vdots$

Derivatives So Far:

- If $f(x) = 1 = x^0$, $f'(x) = 0$
- If $f(x) = x = x^1$, $f'(x) = 1$
- If $f(x) = x^2$, $f'(x) = 2x$
- If $f(x) = \dfrac{1}{x} = x^{-1}$, $f'(x) = -\dfrac{1}{x^2} = -x^{-2}$
- If $f(x) = \sqrt{x} = x^{1/2}$, $f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$
- If $f(x) = x^3$, $f'(x) = 3x^2$

Remember: Pascal's Triangle

1

1 1

1 2 1

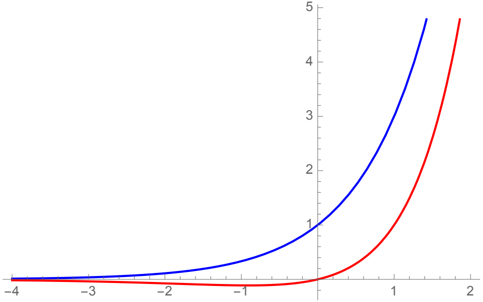
1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 5 1

Is $\frac{d}{dx}(3^x) = x3^{x-1}$?



- Which function is which?
- Based on what you see here, could $\frac{d}{dx}(3^x) = x3^{x-1}$?