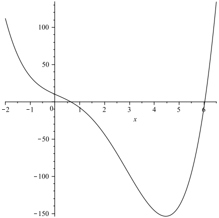
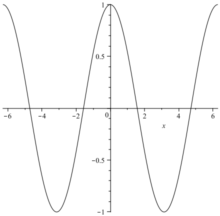
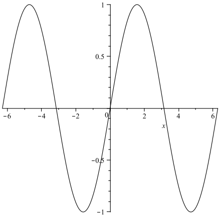
Basic Building Block Functions:



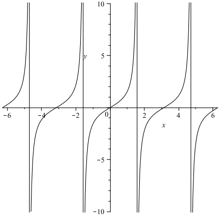
polynomial



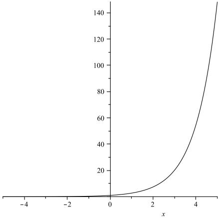
$\cos(x)$



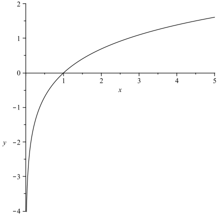
$\sin(x)$



$\tan(x)$



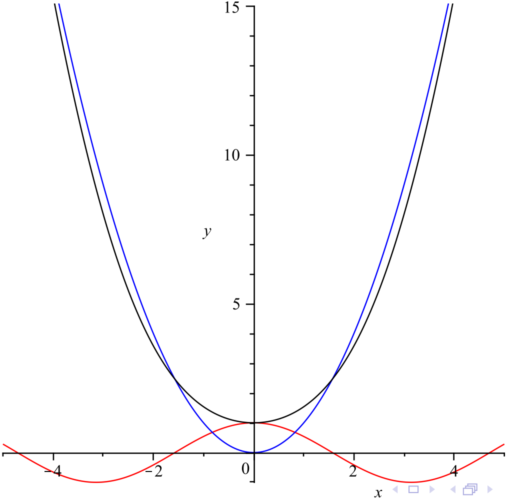
e^x



$\ln(x)$

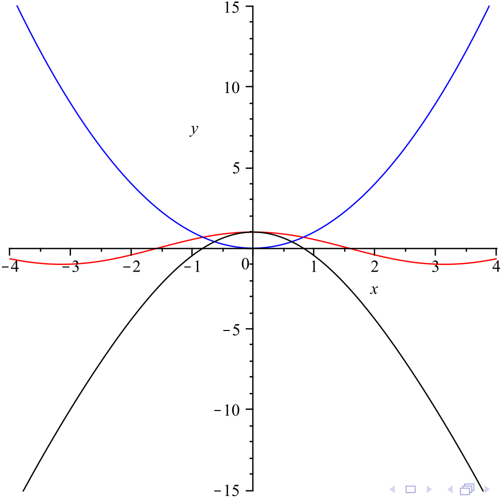
Addition of Functions

If $h(x) = \cos(x)$ and $j(x) = x^2$, then
$(h + j)(x) = h(x) + j(x) = \cos(x) + x^2$



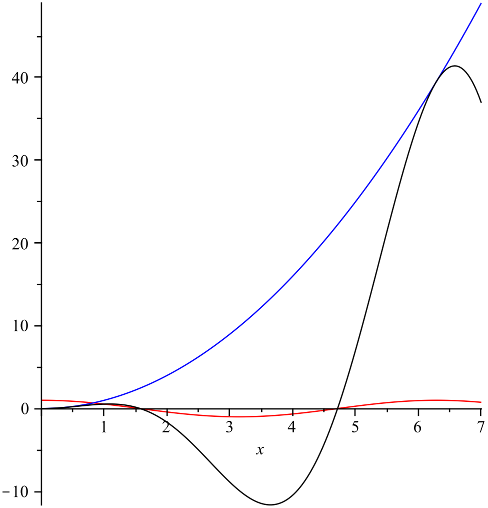
Subtraction of Functions

If $h(x) = \cos(x)$ and $j(x) = x^2$, then
$(h - j)(x) = h(x) - j(x) = \cos(x) - x^2$



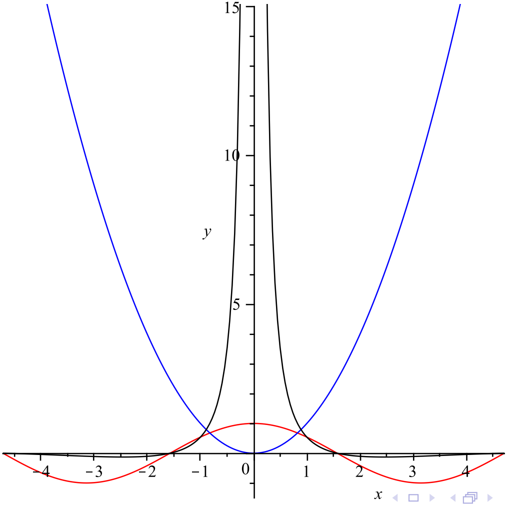
Multiplication of Functions

If $h(x) = \cos(x)$ and $j(x) = x^2$, then $(hj)(x) = h(x)j(x) = x^2\cos(x)$



Division of Functions

If $h(x) = \cos(x)$ and $j(x) = x^2$, then $\frac{h}{j}(x) = \frac{h(x)}{j(x)} = \frac{\cos(x)}{x^2}$



Question:

Let $f(x) = x^2 + 5x$ and $g(x) = \tan(x)$.
(We'll review $\tan(x)$ in a few days.)

1. Find $f \circ g(x)$
2. Find $g \circ f(x)$

Question and Answer:

Let $f(x) = x^2 + 5x$ and $g(x) = \tan(x)$.

(a) Find $f \circ g(x)$

Definition: $f \circ g(x) = f\big(g(x)\big)$

$$f \circ g(x) = f(\tan(x)) = (\tan(x))^2 + 5\tan(x)$$

(b) Find $g \circ f(x)$

$$g \circ f(x) = g(x^2 + 5x) = \tan(x^2 + 5x)$$

Question:

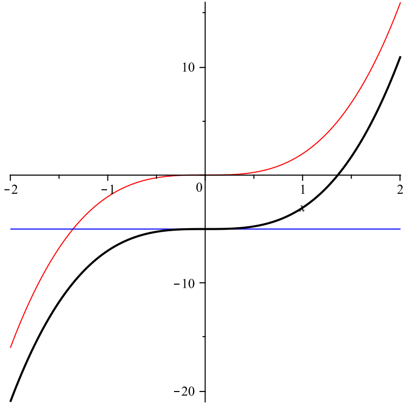
1. Let $g(x) = 2x^3$. How is the graph of $h(x) = g(x) - 5$ related to the graph of $g(x)$, and why?

2. Let $f(x) = \cos(x)$. How is the graph of $g(x) = f(x + \pi/2)$ related to the graph of $f(x)$, and why?

   (We'll review $\cos(x)$ in a few days.)

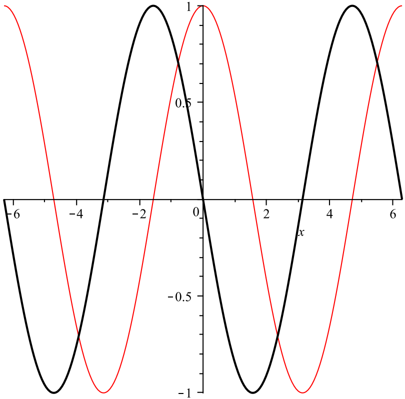
Answer: Vertical Translation

1. If $g(x) = 2x^3$ and $k = -5$, then the graph of $h(x) = g(x) + k = 2x^3 - 5$ is shifted up vertically by k units. In this case, k is negative, so the graph is shifted down 5 units.



Answer: Horizontal Translation

2. If $f(x) = \cos(x)$ and $k = \pi/2$, then the graph of $g(x) = \cos(x + \pi/2)$ is shifted left horizontally by k units.



x	$f(x)$	$g(x)$
0	$\cos(0)$	$\cos(\pi/2)$
$\pi/2$	$\cos(\pi/2)$	$\cos(\pi/2 + \pi/2$
π	$\cos(\pi)$	$\cos(\pi + \pi/2)$
$3\pi/2$	$\cos(3\pi/2)$	$\cos(3\pi/2 + \pi/$

More Questions!

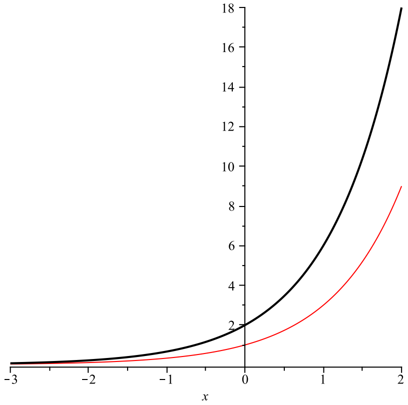
1. Let $f(x) = 3^x$. How is the graph of $g(x) = 2f(x)$ related to the graph of $f(x)$?

   (We'll review exponential functions in a few days.)

2. Let $f(x) = \sin(x)$. How is the graph of $g(x) = f(2x)$ related to the graph of $f(x)$?

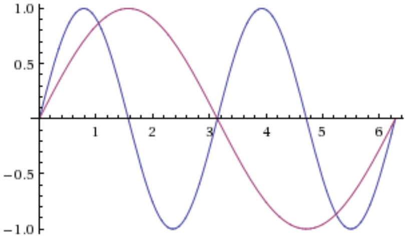
Answer: Vertical Stretching

1. If $f(x) = 3^x$ and $k = 2$, then the graph of $g(x) = 2 \cdot 3^x$ (which is not the same as 6^x) is stretched vertically by a factor of 2: every height $g(a)$ is twice the height $f(a)$.



Answer: Horizontal Compression

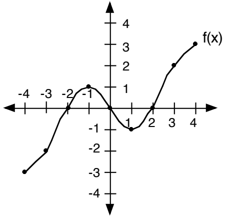
2. If $f(x) = \sin(x)$ and $k = 2$, then the graph of $g(x) = \sin(2x)$ is compressed horizontally by a factor of 2: If the point (x, y) lies on the graph of f, the the point $(x/2, y)$ lies on the graph of g.

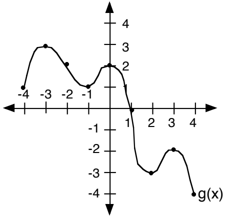


x	$\sin(x)$	$\sin(2x)$
0	$\sin(0)$	$\sin(0)$
$\pi/4$	$\sin(\pi/4)$	$\sin(\pi/2)$
$\pi/2$	$\sin(\pi/2)$	$\sin(\pi)$
$3\pi/4$	$\sin(3\pi/4)$	$\sin(3\pi/2)$
π	$\sin(\pi)$	$\sin(2\pi)$

In Class Work

1. Given $f(x) = 2^x$, $g(x) = \cos(x)$, and $h(x) = x^3 - 2x$, find:
   (a) $f \circ g(x)$
   (b) $f \circ h(x)$
   (c) $h \circ g(x)$
   (d) $f \circ g \circ h(x)$
2. The functions below define $f(x)$ and $g(x)$. Find $f \circ g(-3)$.





3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (a) $f(x)$, $f(x) + 2$, and $f(x) - 3$
   (b) $f(x)$, $f(x+2)$, and $f(x-3)$
   (c) $f(x)$, $2f(x)$, $-3f(x)$, and $\frac{1}{2}f(x)$

Solutions to In Class Work

1. Given $f(x) = 2^x$, $g(x) = \cos(x)$, and $h(x) = x^3 - 2x$, find:
   (a) $f \circ g(x)$

$$f \circ g(x) = f\big(g(x)\big) = f\big(\cos(x)\big) = 2^{\cos(x)}.$$

   (b) $f \circ h(x)$

$$f \circ h(x) = f\big(h(x)\big) = f\big(x^3 - 2x\big) = 2^{x^3-2x}.$$

   (c) $h \circ g(x)$

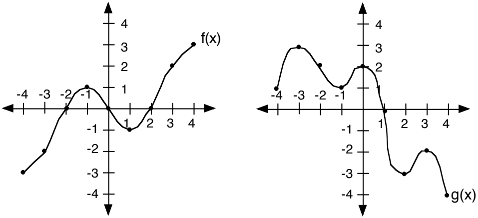
$$h \circ g(x) = h\big(g(x)\big) = h\big(\cos(x)\big) = \big(\cos(x)\big)^3 - 2\cos(x).$$

   (d) $f \circ g \circ h(x)$

$$\begin{aligned} f \circ g \circ h(x) &= f\Big(g\big(h(x)\big)\Big) = f\Big(g\big(x^3 - 2x\big)\Big) = f\Big(\cos(x^3 - 2x)\Big) \\ &= 2^{\cos(x^3-2x)}. \end{aligned}$$

In Class Work

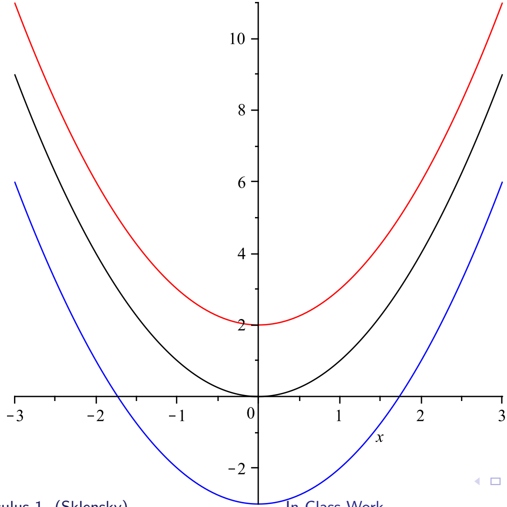
2. The functions below define $f(x)$ and $g(x)$. Find $f \circ g(-3)$.



- $f \circ g(-3) = f\big(g(-3)\big)$.
- From the graph of $g(x)$, $g(-3) = 3$.
- Thus $f \circ g(-3) = f\big(g(-3)\big) = f(3)$
- From the graph of $f(x)$, $f(3) = 2$.
- Thus $f \circ g(-3) = 2$.

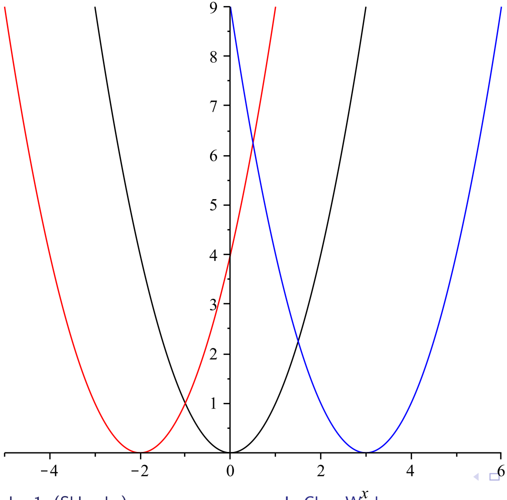
In Class Work

3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (a) $f(x)$, $f(x) + 2$, and $f(x) - 3$



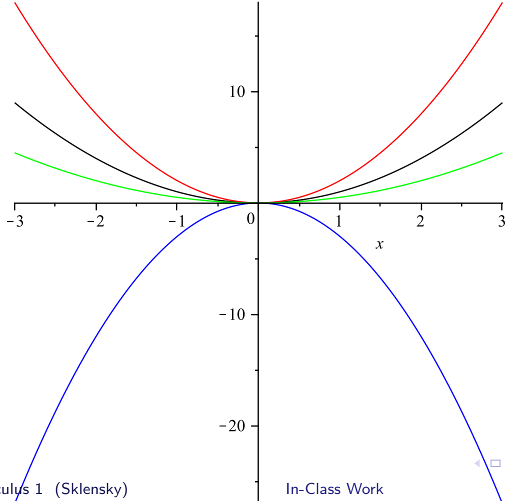
In Class Work

3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (b) $f(x)$, $f(x+2)$, and $f(x-3)$



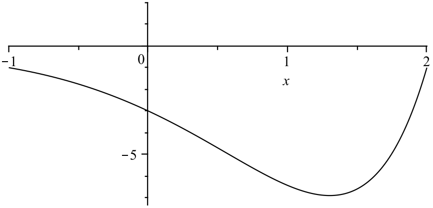
In Class Work

3. Let $f(x) = x^2$. In each case, sketch all listed functions on the same set of axes.
   (c) $f(x)$, $2f(x)$, $-3f(x)$, and $\frac{1}{2}f(x)$



In Class Practice

1. Find the compositions $f \circ g(x)$ and $g \circ f(x)$ if $f(x) = x^2 + x$ and $g(x) = \sin(x)$.
2. Identify functions $f(x)$ and $g(x)$ such that $\sqrt{(x-2)^4 + 3}$ is $f \circ g(x)$.
3. Use the graph below to sketch the graph of $3f(x) + 5$



Solutions:

3. Find the compositions $f \circ g(x)$ and $g \circ f(x)$ if $f(x) = x^2 + x$ and $g(x) = \sin(x)$.
   (a) $f \circ g(x) = f\big(g(x)\big) = f\big(\sin(x)\big) = \sin^2(x) + \sin(x)$
   (b) $g \circ f(x) = g\big(f(x)\big) = g\big(x^2 + x\big) = \sin(x^2 + x)$
4. Identify functions $f(x)$ and $g(x)$ such that $\sqrt{(x-2)^4 + 3}$ is $f \circ g(x)$.
   - One possibility: $g(x) = x - 2$, $f(x) = \sqrt{x^4 + 3}$
   - Another possibiity: $g(x) = (x-2)^4$, $f(x) = \sqrt{x + 3}$
   - Yet another possibility: $g(x) = (x-2)^4 + 3$, $f(x) = \sqrt{x}$.

Solutions:

5. Use the graph below to sketch the graph of $3f(x)+5$