Rolle's Thm & the Mean Value Thm (MVT)

Rolle's Theorem: If $\left\{ \begin{array}{l} f \text{ is continuous on } [a,b] \\ f \text{ is differentiable on } (a,b), \\ \text{and } f(a) = f(b), \end{array} \right\}$ hypotheses

then $\left\{ \text{there is a number } c \in (a,b) \text{ for which } f'(c) = 0. \right\}$ conclusion

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Mean Value Thm: If $\left\{ \begin{array}{l} f \text{ is continuous on } [a,b] \\ f \text{ is differentiable on } (a,b), \end{array} \right\}$ hypotheses

then $\left\{ \begin{array}{l} \text{there is a number } c \in (a,b) \text{ for which} \\ \quad f'(c) = \dfrac{f(b) - f(a)}{b - a} \\ \qquad\qquad = \text{slope of secant btwn } \big(a, f(a)\big), \big(b, f(b)\big) \end{array} \right\}$ conclusion

Hypothoses vs Conclusions

Example: If it is raining, then there are clouds in the sky.

(it is raining: hypothesis; there are clouds in the sky: conclusion)

- Question: If you know it is raining, do you need to look at the sky in order to conclude whether or not there are clouds in the sky?

Hypotheses vs Conclusions

Example: If $\underbrace{\text{it is raining}}_{\text{hypothesis}}$, then $\underbrace{\text{there are clouds in the sky}}_{\text{conclusion}}$.

- Question: If you know it is raining, do you need to look at the sky in order to conclude whether or not there are clouds in the sky?

  No. If the hypothesis is satisfied, the conclusion **must** follow.

- Question: If you know it is **not** raining, does that mean there can not be clouds in the sky?

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- Question: If you know it is **not** raining, does that mean there can not be clouds in the sky?

  No. Hypothesis being false does not force conclusion to be false

- Question: If you know it is **not** raining, do you need to check further to discover whether or not there are clouds in the sky?

Hypotheses vs Conclusions

Example: If $\underbrace{\text{it is raining}}_{\text{hypothesis}}$, then $\underbrace{\text{there are clouds in the sky}}_{\text{conclusion}}$.

- Question: If you know it is raining, do you need to look at the sky in order to conclude whether or not there are clouds in the sky?

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- Question: If you know it is **not** raining, do you need to check further to discover whether or not there are clouds in the sky?

  Yes. Hypothesis being false means need to check further to discover whether or not conclusion is true.

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Mean Value Thm: If $\left\{\begin{array}{l} f \text{ is continuous on } [a,b] \\ f \text{ is differentiable on } (a,b), \end{array}\right\}$ hypotheses

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Known Derivatives and Antiderivatives:

The Derivative

$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
b^x	$\ln(b)b^x$
$\ln(x)$	$\frac{1}{x}$
$\log_b(x)$	$\frac{1}{\ln(b)x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$

An Antiderivative

$f(x)$	$F(x)$
x^n	$\frac{x^{n+1}}{n+1}$, $n \neq -1$
$x^{-1} = \frac{1}{x}$	$\ln(x)$ if $x > 0$
e^x	e^x
b^x	$\frac{b^x}{\ln(b)}$
$\ln(x)$	?
$\log_b(x)$	?
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\tan(x)$	?
$\cot(x)$	?
$\sec(x)$	?
$\csc(x)$	?

Question:

- We know: If $f(x)$ is constant, then $f'(x) = 0$.

  Example: $\frac{d}{dx}(5) = 0$.

- Question:

  Is that the only way $f'(x)$ can equal zero?