DWW Problem 3 (really, #2)

Find the derivative of

$$y = \left(\frac{x^2+2}{3}\right)^5 = \left(\frac{x^2+5}{3}\right)^5.$$

- Easiest way:
  Recognize that the division by 3 is just multiplication by a constant, that is, by 1/3, and so we don't have to use the quotient rule, but can just use the constant multiple rule.
  Outer function: $g(u) = u^5 \Rightarrow g'(u) = 5u^4$
  Inner function: $h(x) = \frac{1}{3}(x^2+5) \Rightarrow h'(x) = \frac{1}{3}(2x) = \frac{2x}{3}$

$$\begin{aligned} y &= g(h(x)) \\ \frac{dy}{dx} &= g'(h(x))h'(x) = 5\left(\frac{x^2+5}{3}\right)^4\left(\frac{2x}{3}\right) \end{aligned}$$

DWW Problem 3 (really, #2)

Find the derivative of

$$y = \left(\frac{x^2+2}{3}\right)^5 = \left(\frac{x^2+5}{3}\right)^5.$$

- Harder way:
  If you don't recognize that you don't have to use the quotient rule on the inner function.
  Outer function: $g(u) = u^5 \Rightarrow g'(u) = 5u^4$
  Inner function: $h(x) = \frac{x^2+5}{3} \Rightarrow h'(x) = \frac{3 \cdot 2x - (x^2+5) \cdot 0}{3^2} = \frac{2x}{3}$

$$\begin{aligned} y &= g(h(x)) \\ \frac{dy}{dx} &= g'(h(x))h'(x) = 5\left(\frac{x^2+5}{3}\right)^4 \left(\frac{2x}{3}\right) \end{aligned}$$

DWW Problem 5 (really, #4)

Suppose that $y = \dfrac{7}{7x-4}$. Find $\dfrac{dy}{dx}$.

- Use the quotient rule on $y = \dfrac{7}{7x-4}$.

$$\frac{dy}{dx} = \frac{\text{hi d' ho - ho d' hi}}{\text{ho ho}} = \frac{(7x-4)\cdot 0 - 7\cdot 7}{(7x-4)^2} = -\frac{49}{(7x-4)^2}$$

OR

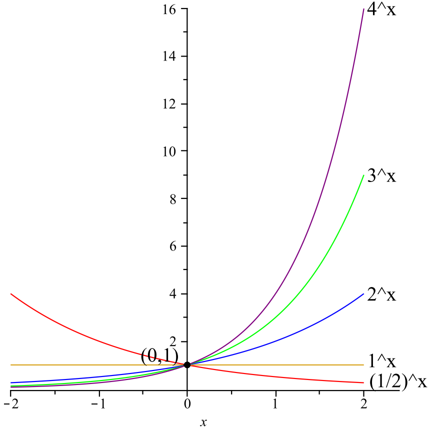
- Use the chain rule on $y = 7\big(7x-4\big)^{-1}$.

$$\frac{dy}{dx} = g'\big(h(x)\big) = 7(-1)\big(7x-4\big)^{-2}(7) = -\frac{49}{(7x-4)^2}$$

Types of Functions We Can't Yet Differentiate

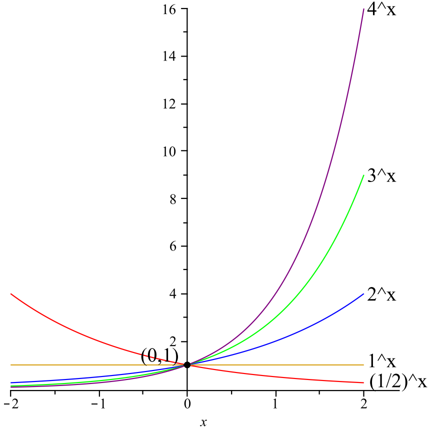
- $f(x) = (x^6 - 14x^5 + 27x^{-3} - 13)(101x^{-1} + 14x^6 + 13 - 42\sqrt{x})$ ✓
- $g(x) = \dfrac{x^7 - \sqrt{x}}{14x^2 + 12}$ ✓
- $h(x) = \left(x^2 + 1\right)^{25}$ ✓
- $j(x) = \cos(x^2)$
- $k(x) = \sin(e^{14x})$
- $m(x) = \ln(\sqrt{x} - 14)$

What IS e?



- Recall: The graph of b^x passes through the point $(0, 1)$ for all b, since $b^0 = 1$.

What IS e?



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- The larger b is, the steeper the slope at $(0,1)$ is.

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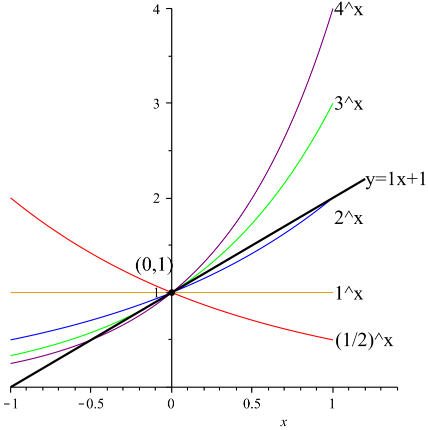

- Recall: The graph of b^x passes through the point $(0,1)$ for all b, since $b^0 = 1$.
- The larger b is, the steeper the slope at $(0,1)$ is.
- If $b \geq 3$, the slope of b^x at $(0,1)$ is larger than 1; if $b \leq 2$, the slope of b^x at $(0,1)$ is less than 1.

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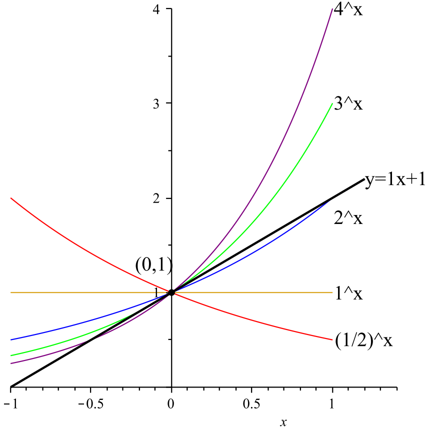

- Recall: The graph of b^x passes through the point $(0, 1)$ for all b, since $b^0 = 1$.
- The larger b is, the steeper the slope at $(0, 1)$ is.
- If $b \geq 3$, the slope of b^x at $(0, 1)$ is larger than 1; if $b \leq 2$, the slope of b^x at $(0, 1)$ is less than 1.
- There is some number b between 2 and 3 for which b^x has slope 1 at $(0, 1)$.

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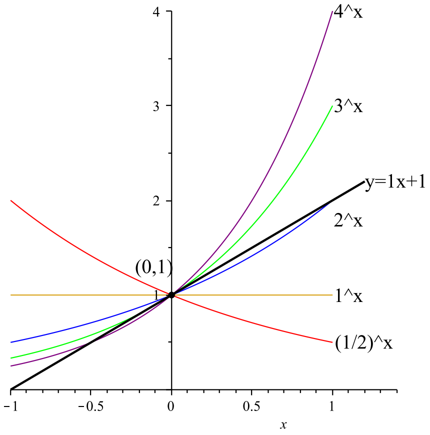

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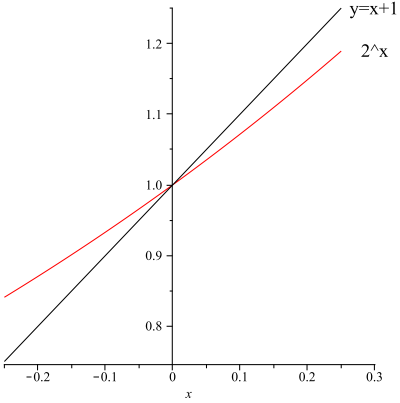
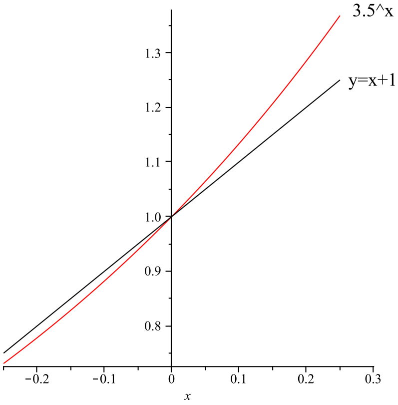
- There is some number b between 2 and 3 for which b^x has slope 1 at $(0,1)$.
- There is some number b btwn 2 and 3 for which line tangent to $y = b^x$ at $(0,1)$ has slope 1.

For what b is slope of b^x at $x = 0$ equal to 1?

That is, for what b is $y = x + 1$ the line tangent to b^x at $(0, 1)$?





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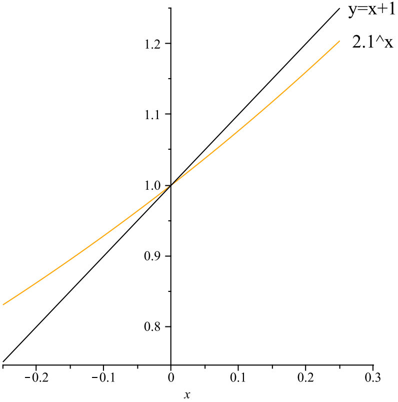

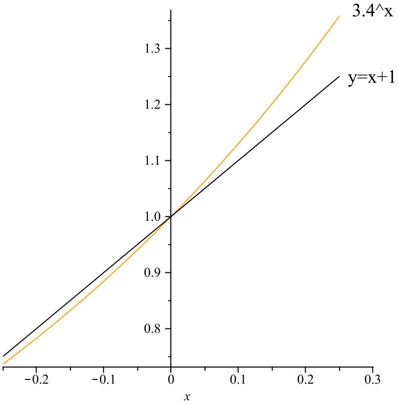

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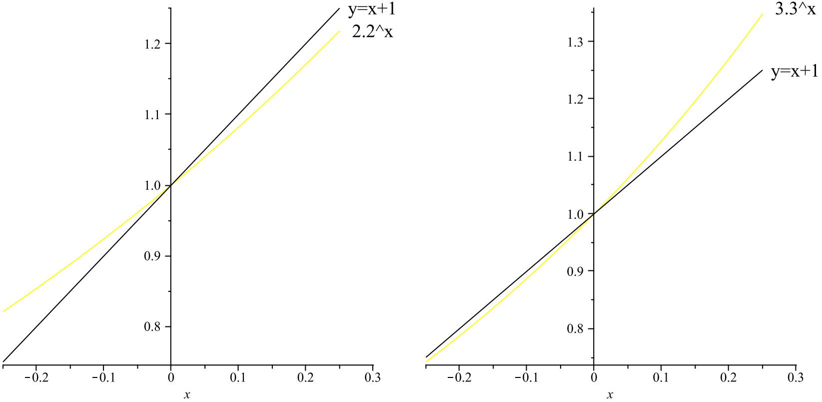

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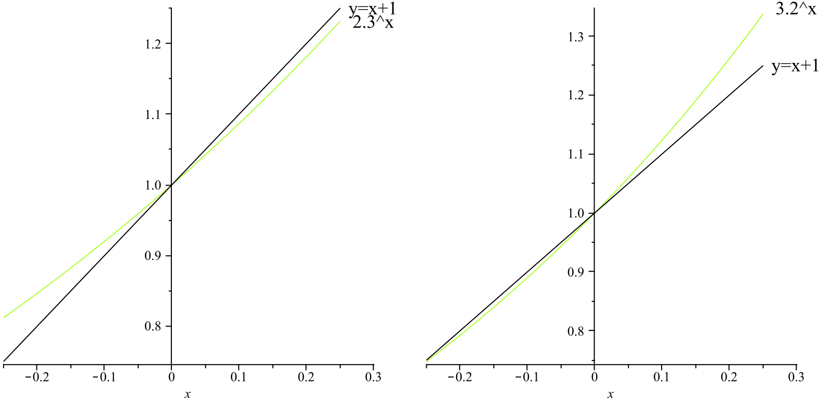

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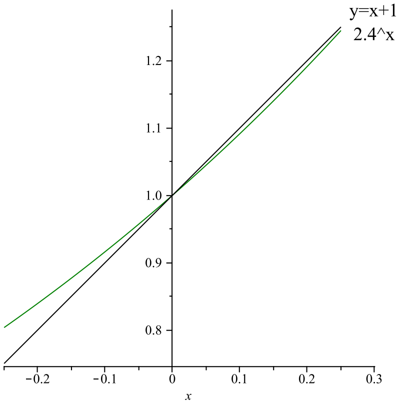

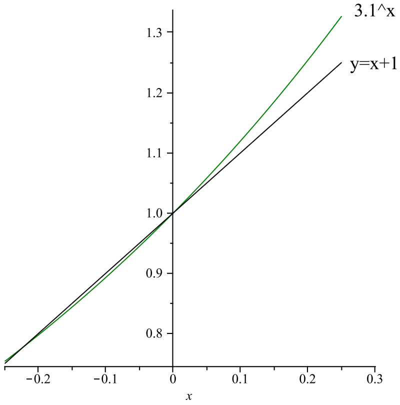

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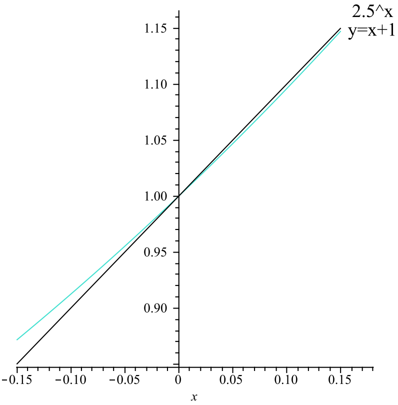
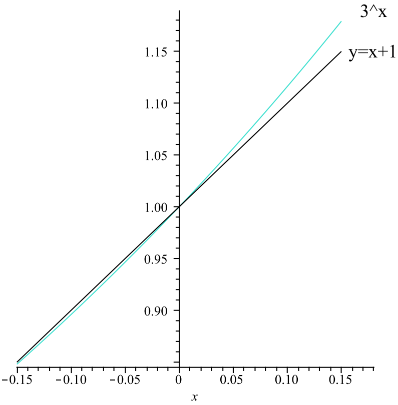
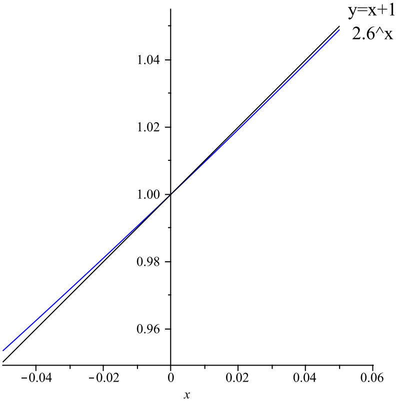
Zoom in!

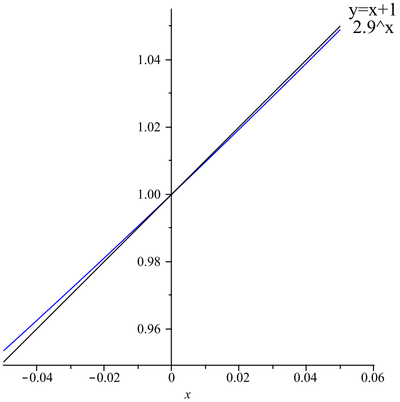




For what b is slope of b^x at $x = 0$ equal to 1?

That is, for what b is $y = x + 1$ the line tangent to b^x at $(0, 1)$?
Zoom in again!





For what b is slope of b^x at $x = 0$ equal to 1?

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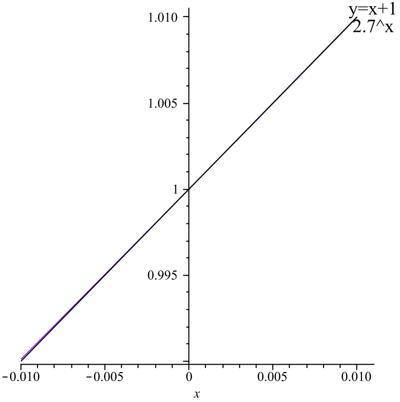
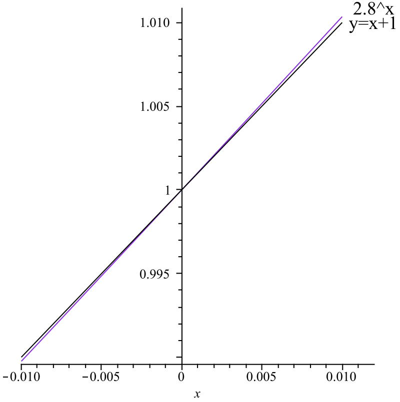
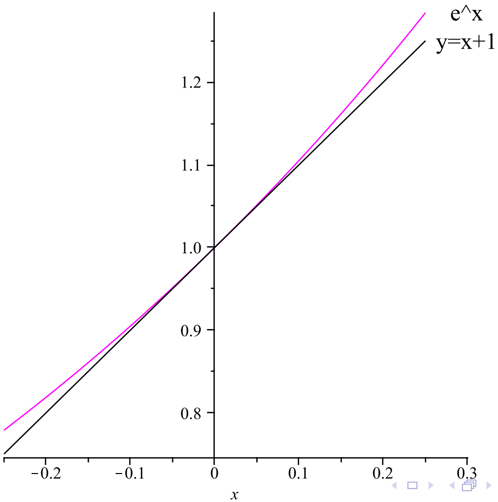
And zoom in one more time!





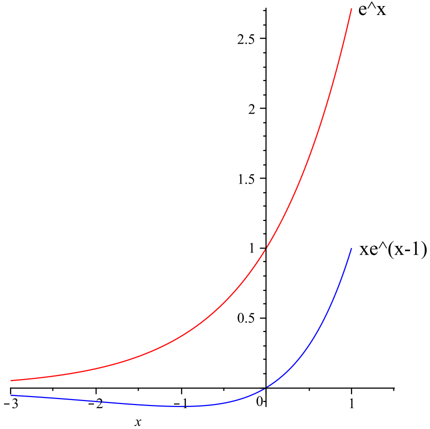
Just the right value of b:

2.71828182845904523536028747135266249775724709369995957496697....



Is $\frac{d}{dx}(e^x) = xe^{x-1}$?

Compare the graphs of e^x and xe^{x-1}:



e^x is always increasing, so $\frac{d}{dx}(e^x)$ should be always positive, but xe^{x-1} is negative for all $x < 0$.

xe^{x-1} is **not** the derivative of e^x.

In Class Work

For each function, find its derivative:

1. $f(x) = 5e^x - 7x^e - 6\ln(10)$

2. $f(x) = (x^2 + 4)3^x$

3. $f(x) = \sqrt{e^x + x}$

4. $f(x) = e^{\sqrt{x}}$

5. $f(x) = e^{\frac{1}{\sqrt{x}+4}}$