Just as with improper integrals,

$$S = S_N + R_N$$

$$\Rightarrow S \approx S_N, \text{ with error } = R_N$$

Or in other words,

$$\sum_{k=m}^{\infty} a_k \approx \sum_{k=m}^{N} a_k, \text{ with error } = \sum_{k=N+1}^{\infty} a_k$$

Similarities and Differences between approximating series and improper integrals:

1. For both, break given expression into a sum of a finite sum or integral plus a tail (or remainder).

2. For both, approximate the given infinite integral or sum using the definite integral or finite sum.

3. For both, make the error however small we want by *bounding the tail.*

4. In the case of improper integrals, we not only approximate the improper integral with a definite one, but we also then have to approximate our definite integral – two sources of error.

   In the case of an infinite sum, once we've dropped the tail, we can just ask Maple to add up the finite number of terms for us, and we'll have our approximation – dropping the tail is the *only* source of error.

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