Consider the sequence  $\left\{\frac{5k^2-42}{3k^2+5}\right\}_{k=1}^{\infty}$ . We want to know whether or not this sequence converges, and if so, what to. Just to try to get a feel for what's going on with this sequence, let's look at the first several terms of this sequence.

 $\begin{array}{c|c|c}
k & a_k \\
\hline
1 & -\frac{37}{8} \\
2 & -\frac{22}{17} \\
3 & \frac{3}{32} \\
4 & \frac{38}{53} \\
5 & \frac{83}{80} \\
6 & \frac{138}{113}
\end{array}$ 

Thus the sequence begins like

$$\{-\frac{37}{8}, -\frac{22}{17}, \frac{3}{32}, \frac{38}{53}, \frac{83}{80}, \frac{138}{113}, \ldots\}$$

Do the following sequences converge or diverge? If the sequence converges, find the limit.

1. 
$$\left\{\frac{j^2 + 32j}{e^j}\right\}_{j=3}^{\infty}$$
  
2. 
$$\left\{\frac{\sin(k)}{k^2}\right\}_{k=1}^{\infty}$$

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**Example:**  $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$  is a geometric series, with  $r = \frac{1}{2}$ .

The associated sequence of terms  $\{a_k\}$  is

$$\left\{ \left(\frac{1}{2}\right)^k \right\}_{k=0}^{\infty} = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \}$$

The associated sequence of partial sums  $S_n$  is

$$\{1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{4}, 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}, \\1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}, \ldots\} = \{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \ldots\}$$

For each series below:

- (a) What is  $a_2$ ?  $a_3$ ?
- (a) What is  $S_2$ ?  $S_3$ ?
- (c) Does the series converge or diverge? If it converges, find the value to which it converges.

$$1. \sum_{k=0}^{\infty} \frac{4}{3^k}$$

 $2. \sum_{k=0}^{\infty} \frac{3^k}{(-4)^k}$ 

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