

1. The  $n$ th Taylor polynomial for  $f(x)$  based at  $x = x_0$  is

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ + \frac{f'''(x_0)}{3!}(x - x_0)^3 \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

2. The idea behind Taylor polynomials approximating a function  $f(x)$  is to focus on how  $f$  behaves at *one point*  $x_0$ . We match not only the  $y$ -values at  $x_0$ , but also the slopes (the first derivative), the concavity (the second derivative), and however many more derivatives we choose –  $n$  is the number of derivatives we're choosing to match.
3. It seems that the higher  $n$  is, the better an approximation  $P_n(x)$  gives.
4. We can approximate  $\cos(x)$  near  $x = 0$  by

$$\cos(x) \approx 1 - x^2/2! + x^4/4! - x^6/6!$$

and near  $x = 2\pi$  by

$$\cos(x) \approx 1 - (x - 2\pi)^2/2! + (x - 2\pi)^4/4! - (x - 2\pi)^6/6!$$