Determining Convergence - Important Reminders

Consider $I = \int_a^\infty f(x)\, dx$.

1. There is a huge distinction between $f(x)$ converging – that is, $\lim_{x\to\infty} f(x)$ being finite – and $I = \int_a^\infty f(x)\, dx$ converging. Just because you can find $\lim_{x\to\infty} f(x)$, and it's a finite number, does **not** mean that $\int_a^\infty f(x)\, dx$ will be finite.

2. In fact, if $\lim_{x\to\infty} f(x)$ exists but is not 0, I diverges! No need to investigate any further.

3. If $\lim_{x\to\infty} f(x) = 0$, I may converge or it may diverge – you must investigate further.

Determine whether each of the following improper integrals converges or diverges.

1. $\displaystyle\int_2^{\infty} \frac{1}{x^3+2}\,dx$

2. $\displaystyle\int_5^{\infty} \frac{1}{\sqrt{x}-2}\,dx$

3. $\displaystyle\int_2^{\infty} \frac{2}{\sqrt{x}+x^2}\,dx$

4. $\displaystyle\int_0^{\infty} \frac{2}{\sqrt{x}+x^2}\,dx$

Goals:

1. Is there any way to at least determine whether or not an improper integral I converges even if we cannot find an antiderivative?

2. Better yet, if we *do* determine that an improper integral I converges, is there a way to approximate the value of the integral I?

Let $I = \int_a^\infty f(x)\, dx$.

Dealing with Goal 1:

1. If $f(x)$ **is** antidifferentiable, cope with I by taking the limit of proper definite integrals. This tells us whether I diverges or converges, and if so, what it converges to.

2. If $f(x)$ is **not** antidifferentiable, then we try to determine whether or not I converges by comparing it to an improper integral whose convergence or divergence we know:

   (a) If I is *less* than or equal to a convergent improper integral (but greater than or equal to 0), it must converge also. If it is *greater than* a convergent improper integral, our comparison was useless.

   (b) If I is *greater* than or equal to a (positive) divergent improper integral, then it must diverge also. If it is *less than* a divergent improper integral, our comparison was useless.

Still left to figure out: Goal 2

If the integrand of an improper integral is *not* antidifferentiable, and you've already determined the improper integral converges, how can you approximate what it converges to?