Goal: Figure out what to do when faced with an improper integral that we can't evaluate simply by taking limits and antidifferentiting.

1. Determine first whether or not the improper integral converges, by comparing it in a useful way to some other integral whose convergence or divergence we know.

2. If the improper integral *does* converge, approximate it.

We know how to do the first step. Now we begin the second step.

Plan for approximating a convergent improper integral:

1. **Replace the improper integral with a proper one**

   Replace $\displaystyle\int_a^\infty f(x)\,dx$ with $\displaystyle\int_a^t f(x)\,dx$.

   The error will be the *tail*, $\displaystyle\int_t^\infty f(x)\,dx$.

   Because the bigger t is, the smaller the tail is, we can control the error introduced by this replacement by making t sufficiently large.

2. **Approximate the proper integral with one of our usual techniques**

   Approximate $\displaystyle\int_a^t f(x)\,dx$ using left, right, midpoint, or trapezoidal sums. This will of course introduce another error.

In each case, determine first whether the improper integral converges or diverges. If it converges, find a definite integral which is within .01 of the improper integral. If it diverges, find a t so that $\displaystyle\int_a^t f(x)\,dx > 100$.

1. $\displaystyle\int_1^\infty \frac{5}{x^3 + e^x}\,dx$
2. $\displaystyle\int_4^\infty \frac{1}{x^{1/2} - x^{1/3}}\,dx$