From Wednesday:

Let $I = \int_0^1 x \sin(x^2)\ dx$

1. Write L_{10} and L_{50} using sigma notation (without using Maple).

   When $n = 10$, $\Delta x = \frac{b-a}{n} = \frac{1-0}{10} = \frac{1}{10}$.

$$\begin{aligned} L_{10} &= \Delta x \sum_{i=0}^{n-1} f(a + i\Delta x) \\ &= \frac{1}{10} \sum_{i=0}^{9} f(\frac{i}{10}) \\ &= \frac{1}{10} \sum_{i=0}^{9} (\frac{i}{10} \sin(\frac{i^2}{100})) \end{aligned}$$

   When $n = 50$, $\Delta x = \frac{b-a}{n} = \frac{1-0}{50} = \frac{1}{50}$.

$$\begin{aligned} L_{50} &= \Delta x \sum_{i=0}^{n-1} f(a + i\Delta x) \\ &= \frac{1}{50} \sum_{i=0}^{49} f(\frac{i}{50}) \\ &= \frac{1}{50} \sum_{i=0}^{49} (\frac{i}{50} \sin(\frac{i^2}{2500})) \end{aligned}$$

2. Write R_{10} and R_{50} using Sigma notation (again, without using Maple).

   The only difference between left and right sums is that rather than going from x_0 to x_{n-1}, we go from x_1 to x_n. In other words, the only difference is that instead of having a rectangle with height $f(a)$ on the left, we have a rectangle with height $f(b)$ on the right.

   Practically speaking, the *only* difference in the appearance of the formulae is that rather than going from $i = 0$ to $n - 1$, we have i going from 1 to n.

$$\begin{aligned} R_{10} &= \frac{1}{10}\sum_{i=1}^{10}\left(\frac{i}{10}\sin\left(\frac{i^2}{100}\right)\right) \\ R_{50} &= \frac{1}{50}\sum_{i=1}^{50}\left(\frac{i}{50}\sin\left(\frac{i^2}{2500}\right)\right) \end{aligned}$$

3. Without calculating any of them, rank I, L_{10} and R_{10} in increasing order.

   Because $x\sin(x^2)$ is increasing, L_{10} under-estimates I while R_{10} over-estimates I. Therefore we have that

$$L_{10} \le I \le R_{10}.$$

4. Can you draw any conclusions about how well L_{10} approximates I?

   Because I is between L_{10} and R_{10}, I is no farther away from L_{10} than R_{10} is from L_{10}. Therefore, even if we can't find the exact value of I, we at least know that the error in using L_{10} to approximate I, which is $|I - L_{10}|$ is no worse than the difference between L_{10} and R_{10}. That is,

$$\text{error} = |I - L_{10}| \le |R_{10} - L_{10}|.$$

5. Use the formal definition of the integral to write $I = \int_0^1 x\sin(x^2)\,dx$ as a limit.

   Using the right sum,

$$\int_0^1 x\sin(x^2)\,dx \stackrel{def}{=} \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\sin(\frac{i^2}{n^2})\right).$$

1. Let $I = \int_0^2 e^{\cos(x)}\ dx$

   (a) Use Maple to calculate L_{40} and R_{40}.
   [Use the leftsum and rightsum commands.]

   What can you conclude about how close these are to the actual value of I?

   (b) Use similar ideas, and trial and error, to find an approximation for I that you *know* is accurate within 0.01.

2. Let $I = \int_0^{\frac{\pi}{2}} x\cos(x)\ dx$

   (a) Calculate T_{40} and M_{40}.

   How close are these to the actual value of I?

   (b) Approximate I accurate within .0001