Inigo Montoya
Thieves' Forest, Florin
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Math 104 Students
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Dear Calculus Students:

HELP ME!! For the last two months, I've had this series of recurring nightmares that are about to drive me out of my mind! Jill Grissom told me how you'd solved her problem, so I thought perhaps you could help me.

You might need to know a little of my background to understand just why this is driving me beyond the brink of insanity. When I was just a young boy, my father was killed by a six-fingered man. I spent the next several decades searching for his killer; after much travail, I finally found and killed him. You'd think all would be blissful now, and when I'm awake indeed life is good. However, as I said, two months ago the nightmares began.

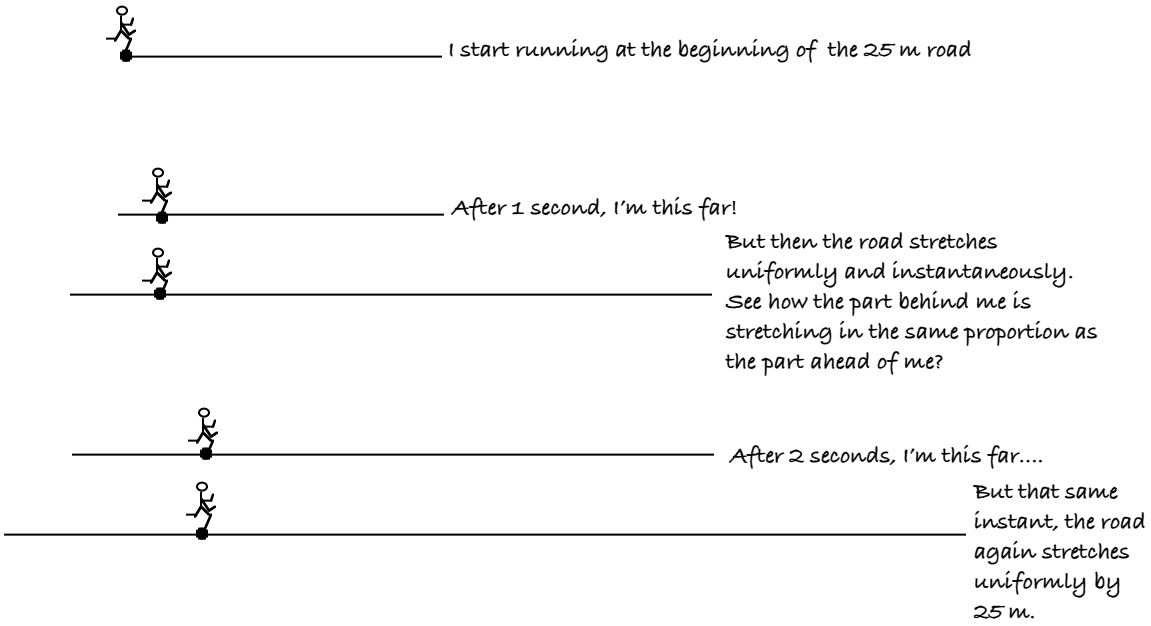
The nightmare is always a variation on the same theme. In the most specific version, I'm standing at the end of a road that I somehow know is exactly 25 meters long. At the other end is my father's six-fingered killer, Count Rugen, looking at me like I'm a bug or something. This peeves me, because I know I already killed him once, and I can do it again. So I say "My name is Inigo Montoya, you killed my father, prepare to die–*again*", and start running toward him. Unfortunately, I can only run in slow motion – in fact, I know I'm running exactly 1 meter per second. That's bad enough, but fairly typical nightmare material so far.

Here's the part that's making me consider a life of insomia! After 1 second, *the road stretches instantaneously* by 25 meters. I try to speed up, but no matter what I do, I'm still moving at that same slow speed of 1 meter per second. After another second, the road stretches again by 25 meters! And this just keeps on happening: I keep running at 1m/sec and every second the road stretches instantaneously by 25m. Over, and over. And over. And over. Well, you get the idea. Then I wake up, frustrated and not exactly refreshed.

The way the road stretches is a little odd. Let me explain. After that first second of running, if the road didn't stretch I'd be just 24 meters away from that six-fingered fiend, right? But the road *does* stretch. However, it stretches uniformly *in both directions*. That is, the new 25m of length isn't added on at one end *or* the other – for instance, it's not like I'm now 49 meters away from the Count. Think of it this way: after that first second, the entire road doubles in length, going from 25m to 50m, right? What I mean by the stretch being uniform is that just like the road doubles in length, the 1 meter that would have been behind me doubles to 2 meters, and the 24m that would have been in front of me doubles to 48m. So after the first second, my tormentor is 48 meters away. It's like I stay where I am, but somebody pulls on both ends of the road at the same time (in a uniform way). I continue running, and after another second the road stretches both behind and in front of me by a total of 25m

again. But this time, since the the road didn't double in length, the parts beyond and ahead of me don't exactly double either; I'm not sure how much they do change by, but I'm sure you can figure it out. They changed proportionally, that's all I can tell you.

Here's a sketch:



I must know: Can I ever get to the six-fingered man? Do I have any chance? If I can get there, how long does it take? Should I dream up a snack to eat along the way?

As I told you earlier, that's just one of **many** variations of my dream. Most of the dreams have the same basic theme, but aren't that specific. Usually, I don't know how long the road is to begin with, or how fast I'm moving. All I know is that I'm always moving at a constant rate, and that the road stretches uniformly and instantaneously by its original amount after each second. Please, **please** help me figure out whether or not I can ever avenge my father (again) in these dreams? Can I save him sometimes, but not others? If and when I **can** save him, how long will it take in each case?

I know you're getting very busy as the semester nears an end, but I'm begging you to help me! Please give me an answer by Tuesday November 20, and please explain it to me clearly enough so that even my subconscious can understand it! I can't take this exhausting torment much longer!

Sleepily yours,

Inigo Montoya

Inigo Montoya

A Few Suggestions from your Professor:

After reading Mr. Montoya's sad tale, I have a couple of suggestions to help you get started.

- Think about *if* Mr. Montoya ever does catch this shady character, will it be the instant before a stretch or the instant after. (I know since it's stretching every second this is a fine distinction, but ponder it.) What has to happen for Mr. Montoya to catch his six-fingered killer? You have to have this clearly in mind to be able to pursue the question of whether or not it's possible.

- Be sure you understand why this six-fingered man is 48 meters away immediately after the first stretch.

- Once you have clear in your mind what Mr. Montoya meant, and what he needs to do, figure out what happens over the first several seconds of his dream. You might want to expand on his sketches.

  For each second you're considering, consider both the situation the moment *before* the stretch and the situation the moment *after* the stretch.

  Carefully label everything. Pay particular attention to those aspects that you've decided are key to discovering whether Mr. Montoya can catch the killer.

- Look for patterns in the lengths you come up with in your sketches. You may see patterns in the lengths themselves, but you're more likely to see patterns in where those lengths came from. That is, you may not want to focus on calculating a bunch of lengths so much as just writing down how you would get the next length from the previous one.

- Figure out a way to describe whatever length you've decided to focus on, k seconds after Mr. Montoya begins running. Generally speaking, it is more useful to have a formula into which you can just plug k and out comes a distance than it is to plug k in and get out a result that depends on $k-1$. If you're initial work gives you this sort of a recursive relationship, work to eliminate it. (This suggestion may not make much sense to you until your run into the situation.)