Find power series expansions about $x_0 = 0$ for the following:

1. $f(x) = \sin(x)$

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

2. $g(x) = \cos(x)$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

3. $h(x) = \cos(x^2)$
   Feel free to use the result from (2).

4. $H(x) = \int \cos(x^2)\, dx$

   Then approximate $\int_0^1 \cos(x^2)\, dx$ accurate within 10^{-5}.

- $\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$
- $\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
- $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

- $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots$

1. Find a power series expansion of $\int_0^1 e^{-x^3}\,dx$. Approximate the value of this integral within 0.001.

2. *A power series for π:*

   2.1 Find a power series expansion for $\frac{1}{1+x^2}$.

   2.2 Find a power series expansion for $\arctan(x)$.

   2.3 Find a power series expansion for $\frac{\pi}{4} = \arctan(1)$.

   2.4 Find a power series expansion for π.