# Reading Question:

*Consider the series* $\sum_{j=0}^{\infty} \frac{1}{2+3^j}$.

1. *Show that the series converges.*

   This series is not geometric, nor is it a p-series, so it is not one whose behavior we already know. The $j$th term test is inconclusive.

   **Comparison Test:** $\frac{1}{2+3^j} \leq \frac{1}{3^j}$ for all $j$, so

   $$\sum_{j=0}^{\infty} \frac{1}{2+3^j} \leq \sum_{j=0}^{\infty} \left(\frac{1}{3}\right)^j = \text{ geometric series with } |r| < 1, \text{ so convergent.}$$

   Thus by the comparison test, our smaller but still non-negative-term series converges.

# Reading Question:

*Consider the series* $\sum_{j=0}^{\infty} \frac{1}{2+3^j}$.

2. *Estimate the limit of the series within* 0.01.
   Need $R_N \leq 0.01$.

$$R_N \stackrel{def}{=} \sum_{j=N+1}^{\infty} \frac{1}{2+3^j} \underbrace{\leq}_{comp} \sum_{j=N+1}^{\infty} \frac{1}{3^j}.$$

   Because

$$\frac{1}{3^{N+1}} \sum_{j=0}^{\infty} \frac{1}{3^j} = \frac{1}{3^{N+1}} \cdot \frac{1}{1-\frac{1}{3}} = \frac{1}{3^{N+1}} \cdot \frac{3}{2} = \frac{1}{2 \cdot 3^N},$$

   if I find $N$ so that $\frac{1}{2 \cdot 3^N} \leq 0.01$, I will have found $N$ so that $R_N \leq 0.01$.

## Reading Question:

*Consider the series* $\displaystyle\sum_{j=0}^{\infty} \frac{1}{2+3^j}$.

2. *Estimate the limit of the series within* 0.01 (continued)

$$\frac{1}{2\cdot 3^N} \le 0.01 = \frac{1}{100} \quad \Rightarrow \quad 2\cdot 3^N \ge 100 \Rightarrow 3^N \ge 50$$

$$\Rightarrow \quad N\ln(3) \ge \ln(50) \Rightarrow N \ge \frac{\ln(50)}{\ln(3)} \approx 3.6$$

Thus $\displaystyle\sum_{j=0}^{4} \frac{1}{2+3^j}$ is within 0.01 of $\displaystyle\sum_{j=0}^{\infty} \frac{1}{2+3^j}$.
Using Maple, I find that $S_4 \approx .67077$, so

$$\sum_{j=0}^{\infty} \frac{1}{2+3^j} \approx .67077 \pm 0.01.$$

# Reading Question:

*Consider the series* $\sum_{j=0}^{\infty} \frac{1}{2+3^j}$.

3. *Is your estimate an over- or under-estimate?*

In calculating $S_4$, I rounded down, so in that sense it's an under-estimate, but that's not what I was really asking.

$S_4$ itself (with no rounding) will be an under-estimate because – since this is a positive-term series – every partial sum (including this one, $S_4$) will be less than the value of the series.

*In Exercises 1-3, determine whether or not the series converges. If it converges, find upper and lower bounds on its limit.*

1. $\sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}}$
2. $\sum_{m=1}^{\infty} \frac{1}{m\sqrt{1 + m^2}}$
3. $\sum_{k=1}^{\infty} \frac{k}{(k^2 + 1)^2}$

*In Exercises 4-6, determine whether or not the series converges or diverges. If the series converges, find a number $N$ such that the partial sum $S_N$ approximate the sum within $10^{-6}$ and then find $S_N$. If the series diverges, find a number $N$ such that $S_N \geq 1000$.*

1. $\sum_{j=1}^{\infty} \frac{1}{100 + 5j}$
2. $\sum_{k=0}^{\infty} \frac{k}{k^6 + 17}$
3. $\sum_{m=2}^{\infty} \frac{\ln(m)}{m^3}$