

Recall:

1. **Definition:** An **alternating series** is one whose terms alternate in sign. That is, a series of the form $a_1 - a_2 + a_3 - \cdots$
where a_k is positive for all $k \geq 1$.

2. **Alternating Series Test:**

- ▶ Suppose $\lim_{k \rightarrow \infty} a_k = 0$ and $\{a_k\}$ is a non-negative decreasing series for

all $k \geq 1$. Then the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ converges.

- ▶ Furthermore, its limit lies between any two consecutive partial sums. That is, if the series converges to S , then S lies between S_N and S_{N+1} for any N .

Because of this,

$$|S - S_N| \leq a_{N+1}.$$

Determine whether or not the following alternating series converge. For those that converge, first find upper and lower bounds, and then approximate accurate to within 0.001.

1.
$$\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln(k)}$$

2.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^2 - 1}$$

3.
$$\sum_{j=3}^{\infty} \frac{(-1)^j}{4^j}$$

Determine whether or not the following alternating series converge conditionally, converge absolutely, or diverge.

1.
$$\sum_{j=0}^{\infty} (-1)^j \frac{e^j}{3^{j+1} + j}$$

2.
$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2k + 2}{k^2 + 2k}$$