1. Use the ratio test to decide whether the following alternating series convergence absolutely, converge conditionally, or diverge:

   1.1 $\displaystyle\sum_{k=12}^{\infty} \frac{(-10)^k}{k!}$

   Since none of the terms are ever zero, the ratio test applies.

$$L = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \left| \frac{(-10)^{k+1}}{(k+1)!} \cdot \frac{k!}{(-10)^k} \right| = \lim_{k\to\infty} \frac{10}{k+1} = 0.$$

   Since $L < 1$, the ratio test tells us this series **converges**!

1.(b) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^{50}}$

Again, for all $n \geq 1$, the terms are non-zero, so the ratio test applies. Whether or not it's conclusive remains to be seen.

Let

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(-2)^{n+1}}{(n+1)^{50}} \cdot \frac{n^{50}}{(-2)^n} \right| = \lim_{n \to \infty} 2 \left(\frac{n}{n+1} \right)^{50} = 2.$$

Since $L > 1$, the ratio test tells us that this series **diverges.**

2. Determine the convergence or divergence of

(a) $\displaystyle\sum_{j=0}^{\infty} \frac{j!}{(j+2)!}$

- This is not at first glance a series we already know.
- We can see the jth term test is inconclusive, if we simplify the terms:

$$\frac{j!}{(j+2)!} = \frac{j(j-1)(j-2)\cdots 3\cdot 2\cdot 1}{(j+2)(j+1)j(j-1)(j-2)\cdots 3\cdot 2\cdot 1} = \frac{1}{(j+2)(j+1)}.$$

- **Ratio Test?** Think of ratio test because of the factorials:

$$\left|\frac{a_{j+1}}{a_j}\right| = \frac{(j+1)!}{(j+3)!}\cdot\frac{(j+2)!}{j!} = \frac{(j+1)!}{j!}\cdot\frac{(j+2)!}{(j+3)!} = \frac{j+1}{j+3}.$$

Thus $\displaystyle\lim_{j\to\infty}\left|\frac{a_{j+1}}{a_j}\right| = \lim_{j\to\infty}\frac{j+1}{j+3} = 1$, so the ratio test is **inconclusive.**

2(a) $\sum_{j=0}^{\infty} \frac{j!}{(j+2)!}$ (continued)

- **Comparison Test?** We found that $\sum_{j=0}^{\infty} \frac{j!}{(j+2)!} = \sum_{j=0}^{\infty} \frac{1}{(j+2)(j+1)}$.

  Want to compare to $\sum \frac{1}{j^2}$, but this can't begin with $j = 0$.

  Adjust for this by pulling out the 0th term in original sum:

$$\begin{aligned}\sum_{j=0}^{\infty} \frac{j!}{(j+2)!} &= a_0 + \sum_{j=1}^{\infty} \frac{1}{(j+2)(j+1)} = \frac{1}{2} + \sum_{j=1}^{\infty} \frac{1}{(j+2)(j+1)} \\ &\leq \frac{1}{2} + \sum_{j=1}^{\infty} \frac{1}{j^2}, \text{ which converges } (p = 2)\end{aligned}$$

  Thus the smaller series $\sum_{j=0}^{\infty} \frac{j!}{(j+2)!}$ converges , by the comparison test.

2(b) $\sum_{n=0}^{\infty} \frac{n^3}{n!}$

- This is not a series we recognize.
- For the nth term test, look at the terms:

$$\frac{n^3}{n!} = \frac{n \cdot n \cdot n}{n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1}.$$

For small n, n^3 may exceed $n!$. But when n gets large, we've got

$$\frac{n^3}{n!} = \frac{n}{n} \cdot \frac{n}{n-1} \frac{n}{n-2} \cdot \frac{1}{(n-3)(n-4) \cdots 3 \cdot 2 \cdot 1}$$

The first three terms approach 1, and the last term approaches 0. Thus the nth term test is inconclusive.

2(b) $\sum_{n=0}^{\infty} \frac{n^3}{n!}$ (continued)

- **Ratio Test?** Again, with the factorials, we first think ratio test.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} = \lim_{n\to\infty} \left(\frac{n+1}{n} \right)^3 \frac{1}{n+1} = 1 \cdot 0 = 0 < 1.$$

Since $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$, the series $\sum_{n=0}^{\infty} \frac{n^3}{n!}$ converges, by the ratio test.