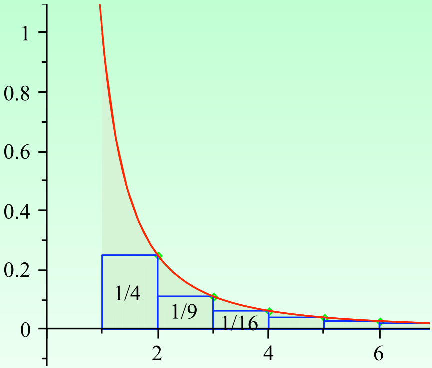
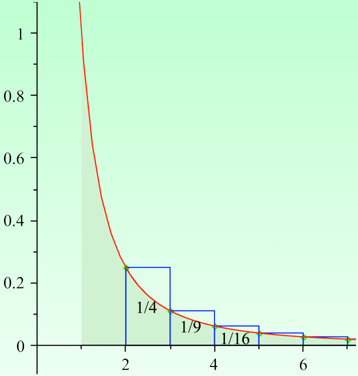
1. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2} = \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$

Represent this as a Riemann Sum for $\displaystyle\int_a^{\infty} \frac{1}{x^2}\, dx$:

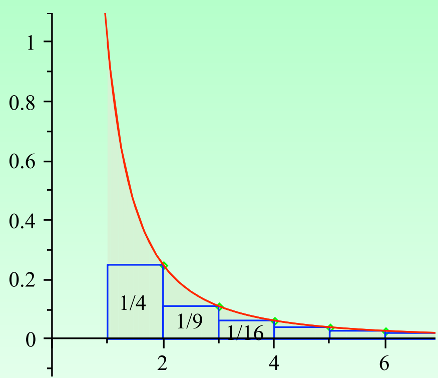


$\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2}$ as a right sum

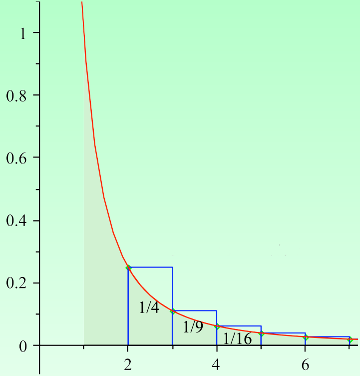


$\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2}$ as a left sum

1. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2} = \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$



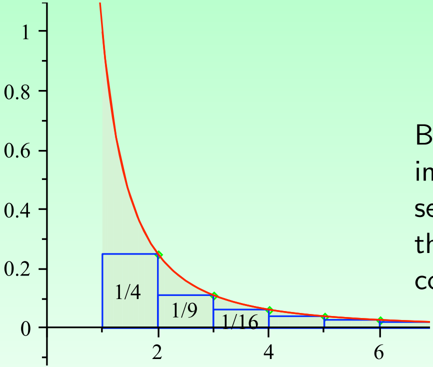
$\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2}$ as a right sum



$\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2}$ as a left sum

Because $\displaystyle\int_a^{\infty} \frac{1}{x^2}\,dx$ converges $(a > 0)$, a sum that is **less** than the integral will also converge.

1. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2} = \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$



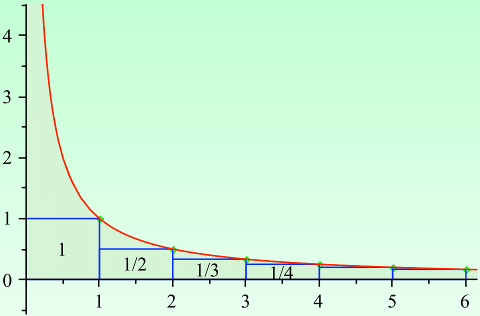
$$\sum_{k=2}^{\infty} \frac{1}{k^2} \leq \int_1^{\infty} \frac{1}{x^2}$$

Because $p = 2$, the improper integral converges and so the series, which can not exceed the improper integral, must also converge.

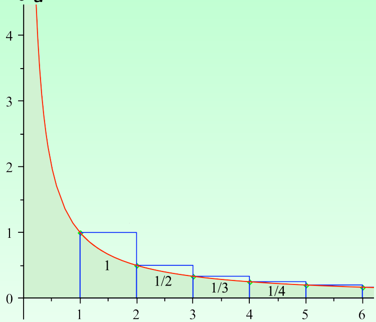
Conclusion: $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2}$ **converges**.

1. $\sum_{k=1}^{\infty} \frac{1}{k}$

Represent this as a Riemann Sum for $\int_a^{\infty} \frac{1}{x}\, dx$:

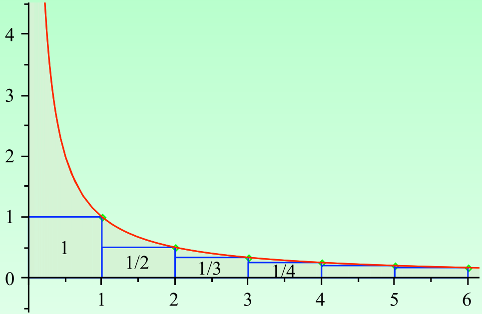


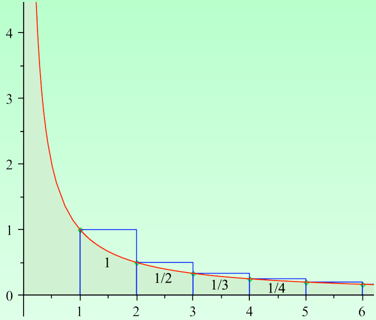
$\sum_{k=1}^{\infty} \frac{1}{k}$ as a right sum



$\sum_{k=1}^{\infty} \frac{1}{k}$ as a left sum

1. $\sum_{k=1}^{\infty} \frac{1}{k}$





$\sum_{k=1}^{\infty} \frac{1}{k}$ as a right sum

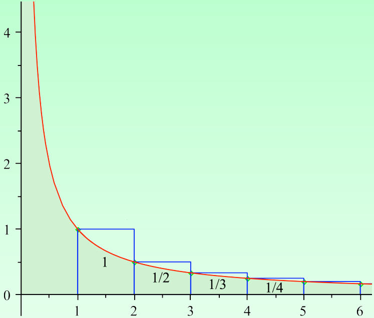
$\sum_{k=1}^{\infty} \frac{1}{k}$ as a left sum

Because $\int_a^{\infty} \frac{1}{x}\, dx$ diverges ($a > 0$), a sum which is larger will also diverge.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$

$$\sum_{k=1}^{\infty} \frac{1}{k} \geq \int_1^{\infty} \frac{1}{x}$$

Because $p = 1$, the improper integral diverges and so the series, which is at least as large as the improper integral, must also diverge.



Conclusion: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ **diverges**.

Goals: Be able to :

1. determine whether a series $\sum a_k$ converges or diverges.
2. If it converges, find the limit (that is, the value of the series) exactly, if possible.
3. If it converges but we can't find the limit exactly, be able to approximate it.