Remember:

- There is a difference between the *sequence of terms* $\{a_k\}$ converging and the *sequence of partial sums* $\{S_n\}$ converging.
- The n-th term test says that if the terms $\{a_k\}$ converge to anything other than 0, or don't converge at all, then there's no way that the partial sums $\{S_n\}$ can converge.
- It does not tell us anything about what happens if the nth term *does* converge to 0.
- Example: The sequence $\left\{\frac{1}{k}\right\}$ converges to 0, but the harmonic series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ diverges.

Integral Test for Positive-Term Series:

Suppose that for all $x \geq 1$, $a(x)$ is continuous, non-negative, and decreasing. Let $a_k = a(k)$ for all $k = 1, 2, \ldots$.

Then the series $\displaystyle\sum_{k=1}^{\infty} a_k$ and the integral $\displaystyle\int_1^{\infty} a(x)\,dx$ either *both* converge or *both* diverge.

The relationship is:

$$\int_1^{\infty} a(x)\,dx \leq \sum_{k=1}^{\infty} a_k \leq a_1 + \int_1^{\infty} a(x)\,dx$$

Immediate Consequence of the Integral Test:

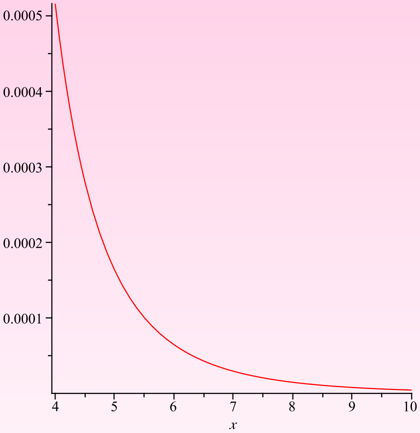
Since $\frac{1}{x^p}$ is a continuous, positive, and decreasing function on $[a, \infty)$ for all $a > 0$ and for all $p > 0$, the integral test applies to $a(x) = \frac{1}{x^p}$ and $\sum_{k=1}^{\infty} \frac{1}{k^p}$ for all $p > 0$.

Thus $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges whenever $\int_1^{\infty} \frac{1}{x^p}\, dx$ does, and diverges whenever the integral does.

So ...

$$\sum_{k=1}^{\infty} \frac{1}{k^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Such series are called **p series**.



$$\text{Graph of } a(x) = \frac{2x^3}{(2x^4 - 14)^2} \text{ on } [4, \infty)$$

Use the integral test to determine whether the following series converge or diverge.

1. $\displaystyle\sum_{j=3}^{\infty} \frac{18j^2}{3j^3+1}$

2. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

You've shown that $\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$ converges.

Let $S = \sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$.

1. Use the integral test to find lower and upper limits for the value of S.

2. Find a number N such that the partial sum S_N approximates the sum of the series within 0.001.