1. You've shown that $\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$ converges.

Let $S = \sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$.

1.1 Use the integral test to find lower and upper limits for the value of S.

Integral test $\Longrightarrow \int_1^{\infty} a(x)\, dx \leq \sum_{k=1}^{\infty} a_k \leq 1 + \int_0^{\infty} a(x)\, dx$.

Since we found that $\int_1^{\infty} xe^{-x^2}\, dx = \frac{1}{2e}$,

$$\begin{aligned} \int_1^{\infty} xe^{-x^2}\, dx \quad &\leq S \leq \quad a_1 + \int_1^{\infty} xe^{-x^2}\, dx \\ \frac{1}{2e} \quad &\leq S \leq \quad \frac{1}{e} + \frac{1}{2e} = \frac{3}{2e} \\ 0.184 \quad &\leq S \leq \quad 0.552 \end{aligned}$$

1.2 Find a number N such that the partial sum S_N approximates the sum of the series within 0.001.

Need $R_N = \displaystyle\sum_{k=N+1}^{\infty} \leq 0.001$.

Since $R_N \leq \displaystyle\int_N^\infty a(x)\,dx$, suffices to find N so $\displaystyle\int_N^\infty a(x)\,dx \leq 0.001$.

$$
\begin{aligned}
\int_N^\infty xe^{-x^2}\,dx &\leq 0.001 \\
-\frac{1}{2}\lim_{R\to\infty} e^{-x^2}\Big|_N^R &\leq 0.001 \\
\frac{e^{-N^2}}{2} &\leq 0.001 \\
e^{-N^2} &\leq 0.002 \\
-N^2 &\leq \ln(0.002) \\
N^2 &\geq -\ln(0.002) \\
N &\geq \sqrt{-\ln(0.002)} = 2.49
\end{aligned}
$$