Methods we have so far:

Given a series $\sum_{k=M}^{\infty} a_k$,

- **Is it a geometric series?** If so, we can determine whether or not it converges, and if so, exactly what it converges to.
- **kth Term Test:** If $\lim_{k\to\infty} a_k \neq 0$, the series diverges. If $\lim_{k\to\infty} a_k = 0$, inconclusive.
- **Integral Test: For non-negative term series:** The series does whatever the associated integral $\int_M^{\infty} a(x)\,dx$ does. If the series converges, we can use $R_N \leq \int_N^{\infty} a(x)\,dx$ to approximate the series to any desired degree of accuracy.
- **Comparison Test: For non-negative term series:** Compare to a larger convergent series or to a smaller divergent series. ...

Suppose that $0 \leq a_k \leq b_k$ for all $k \geq m$.
Then

- If $\sum_{k=m}^{\infty} b_k$ converges, then $\sum_{k=m}^{\infty} a_k$ converges as well, and
  $0 \leq \sum_{k=m}^{\infty} a_k \leq \sum_{k=m}^{\infty} b_k$.
- If $\sum_{k=m}^{\infty} a_k$ diverges, then $\sum_{k=m}^{\infty} b_k$ diverges as well.

Note: As you'll discuss some in your next P.S., what the first $N-1$ terms do doesn't matter, so we could modify this:
Suppose that $0 \leq a_k \leq b_k$ for all $k \geq N \geq m$.
Then

1. If $\sum_{k=m}^{\infty} b_k$ converges, then so does $\sum_{k=m}^{\infty} a_k$.
2. If $\sum_{k=m}^{\infty} a_k$ diverges, then so does $\sum_{k=m}^{\infty} b_k$.

Determine the convergence or divergence of the following three series:

1. $\displaystyle\sum_{k=2}^{\infty} \frac{3^k}{5^k + 2k}$
2. $\displaystyle\sum_{k=2}^{\infty} \frac{2k}{7k + 18}$
3. $\displaystyle\sum_{j=5}^{\infty} \frac{j!}{(j+2)!}$

Recall:

For a **non-negative-term** series,

$$\begin{aligned}
\sum_{k=m}^{\infty} a_k &= \sum_{k=m}^{N} a_k + \sum_{k=N+1}^{\infty} a_k \\
\Rightarrow \sum_{k=m}^{\infty} a_k &\approx \sum_{k=m}^{N} a_k, \text{ with error } = \sum_{k=N+1}^{\infty} a_k
\end{aligned}$$

$$\begin{aligned}
\text{or in other words, } S &= S_N + R_N \\
\Rightarrow S &\approx S_N, \text{ with error } = R_N
\end{aligned}$$

Consider the series $\sum_{k=0}^{\infty} \frac{1}{k+2^k}$.

1. Use the comparison test to show that $\sum_{k=0}^{\infty} \frac{1}{k+2^k}$ converges.

2. Find an N so that $R_N \leq 0.001$.

3. Compute an estimate of the limit of the series that is guaranteed to be within 0.001 of the exact value.

4. Is your estimate in part (c) an overestimate or an underestimate?