Using $P_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \cdots + a_nx^n$, where $a_i = \dfrac{f^{(i)}(0)}{i!}$, the 6th Taylor polynomial for $\cos(x)$ based at $x = 0$ is

$$\cos(x) \approx P_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}.$$

Remember: Idea behind Taylor polynomials is that the function and the derivatives of a Taylor polynomial at the base point (here, $x_0 = 0$) should match the original function (here, $\cos(x)$) and its derivatives at the base point.

Do they?

$$f(x) = \cos(x)$$
$$P_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$\cos(x)$ and derivatives at $x_0 = 0$		$P_6(x)$ and derivatives at $x_0 = 0$	
$f(x) = \cos(x)$	$f(0) = 1$	$P_6(x)$	$P_6(0) = 1$
$f'(x) = -\sin(x)$	$f'(0) = 0$	$P_6'(x) = -x + \frac{x^3}{3!} - \frac{x^5}{5!}$	$P_6'(0) = 0$
$f''(x) = -\cos(x)$	$f''(0) = -1$	$P_6''(x) = -1 + \frac{x^2}{2} - \frac{x^4}{4!}$	$P_6''(0) = -1$
$f'''(x) = \sin(x)$	$f'''(0) = 0$	$P_6'''(x) = x - \frac{x^3}{3!}$	$P_6'''(0) = 0$
$f^{(4)}(x) = \cos(x)$	$f^{(4)}(0) = 1$	$P_6^{(4)}(x) = 1 - \frac{x^2}{2}$	$P_6^{(4)}(0) = 1$
$f^{(5)}(x) = -\sin(x)$	$f^{(5)}(0) = 0$	$P_6^{(5)}(x) = -x$	$P_6^{(5)}(0) = 0$
$f^{(6)}(x) = -\cos(x)$	$f^{(6)}(0) = -1$	$P_6^{(6)}(x) = -1$	$P_6^{(6)}(0) = -1$

After the 6th, the derivs of $\cos(x)$ and $P_6(x)$ no longer *must* match, although in fact, all the odd derivatives *will* match (they'll both be 0).

Recall: if $a_i = \dfrac{f^{(i)}(x_0)}{i!}$,

$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + a_3(x - x_0)^3 \cdots + a_n(x - x_0)^n$,

Let $f(x) = e^x$.

(a) Find $P_1(x)$, $P_2(x)$, $P_3(x)$, and $P_4(x)$ for $f(x)$ based at $x_0 = 0$

(b) Graph e^x, $P_1(x)$, $P_2(x)$, $P_3(x)$ and $P_4(x)$ all on the same set of axes. Find intervals on which each Taylor polynomial is a good approximation.

(c) Approximate $e^{-1/2}$ using $P_4(x)$. Based on the graph, will that be an over- or under-estimate? Compare to what Maple gives for $e^{-1/2}$.