Evaluate the following integrals using integration by parts, and *check your answers!!*

1. $\int xe^x \, dx$

$$\begin{array}{ll} u = x & dv = e^x \, dx \\ du = dx & v = e^x \end{array}$$

$$\int xe^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x + C$$

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**Verify:** $\dfrac{d}{dx}(xe^x - e^x + C) = xe^x + e^x - e^x = xe^x$

2. $\int x \ln(x)\, dx$

$$u = \ln(x) \qquad dv = x\, dx$$

$$du = \frac{1}{x}\, dx \qquad v = \frac{x^2}{2}$$

$$\begin{aligned}\int x \ln(4x)\, dx = uv - \int v\, du &= \frac{x^2}{2}\ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x}\, dx \\ &= \frac{x^2 \ln(x)}{2} - \frac{1}{2}\int x\, dx \\ &= \frac{x^2 \ln(x)}{2} - \frac{1}{2} * \frac{x^2}{2} + C\end{aligned}$$

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**Verify:**

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C\right) &= \frac{1}{2} * x^2 * \frac{1}{x} + \frac{1}{2}\ln(x) * 2x - \frac{1}{4} * 2x \\ &= \frac{x}{2} + x \ln(x) - \frac{x}{2}\end{aligned}$$

3. $\int_0^1 x \sin(\pi x)\ dx$

$$u = x \qquad dv = \sin(\pi x)\ dx$$
$$du = dx \qquad v = -\tfrac{1}{\pi}\cos(\pi x)$$

$$\begin{aligned}
\int_0^1 x\sin(\pi x)\ dx &= uv - \int v\ du \\
&= \left(-\frac{x}{\pi}\cos(\pi x) + \frac{1}{\pi}\int \cos(\pi x)\ dx\right)\Bigg|_0^1 \\
&= \left(-\frac{x}{\pi}\cos(\pi x) + \frac{1}{\pi^2}\sin(\pi x)\right)\Bigg|_0^1 \\
&= \left(-\frac{1}{\pi}\cos(\pi) + \frac{1}{\pi^2}\sin(\pi)\right) - \left(0 + \frac{1}{\pi^2}\sin(0)\right) \\
&= -\frac{1}{\pi}\cdot -1 + 0 \\
&= \frac{1}{\pi}
\end{aligned}$$

(No room to verify on this slide but you should do it!)

4. $\int x^2 \cos(2x)\, dx$

$$u = x^2 \qquad dv = \cos(2x)\, dx$$
$$du = 2x\, dx \qquad v = \frac{1}{2}\sin(2x)$$

$$\int x^2 \cos(2x)\, dx = uv - \int v\, du = \frac{x^2}{2}\sin(2x) - \int x \sin(2x)\, dx$$

$$u = x \qquad dv = \sin(2x)\, dx$$
$$du = dx \qquad v = -\tfrac{1}{2}\cos(2x)$$

$$\begin{aligned}
\int x^2 \cos(2x)\, dx &= \frac{x^2}{2}\sin(2x) - [uv - \int v\, du] \\
&= \frac{x^2}{2}\sin(2x) - [-\frac{x}{2}\cos(2x) + \frac{1}{2}\int \cos(2x)\, dx] \\
&= \frac{x^2}{2}\sin(2x) + \frac{x}{2}\cos(2x) - \frac{1}{2} * \frac{1}{2}\sin(2x) + C \\
&= (\frac{x^2}{2} - \frac{1}{4})\sin(2x) + \frac{x}{2}\cos(2x) + C
\end{aligned}$$

5. $\int x \sec^2(4x)\ dx$

$u = x \qquad dv = \sec^2(4x)\ dx$

$du = dx \qquad v = \frac{1}{4}\tan(4x)$

$$\begin{aligned}
\int x \sec^2(4x)\ dx &= uv - \int v\ du \\
&= \frac{1}{4}\left(x\tan(4x) - \int \tan(4x)\ dx\right) \\
&= \frac{1}{4}\left(x\tan(4x) - \int \frac{\sin(4x)}{\cos(4x)}\ dx\right) \\
&= u = \cos(4x) \Rightarrow -\frac{1}{4}\ du = \sin(4x)\ dx \\
&= \frac{1}{4}\left(x\tan(4x) + \frac{1}{4}\int \frac{1}{u}\ du\right) \\
&= \frac{x}{4}\tan(4x) + \frac{1}{16}\ln|\cos(4x)| + C
\end{aligned}$$

6. $\displaystyle\int \ln(x)\,dx$

$$\begin{array}{ll} u = \ln(x) & dv = dx \\ du = \frac{1}{x}\,dx & v = x \end{array}$$

$$\begin{aligned} \int \ln(x)\,dx &= uv - \int v\,du \\ &= x\ln(x) - \int x \cdot \frac{1}{x}\,dx \\ &= x\ln(x) - \int 1\,dx \\ &= x\ln(x) - x + C \end{aligned}$$

Verify:

$$\begin{aligned} \frac{d}{dx}(x\ln(x) - x + C) &= x \cdot \frac{1}{x} + \ln(x) - 1 \\ &= \ln(x) \end{aligned}$$