Theorem (Special Case): The $n$thTaylor Polynomial for $f(x)$ based at $x = 0$ (a.k.a. the nth Maclaurin polynomial) is given by
$P_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \cdots + a_nx^n$, where $a_i = \dfrac{f^{(i)}(0)}{i!}$.

Theorem (General Case): The nth Taylor polynomial for $f(x)$ based at $x = x_0$ is given by
$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + a_3(x - x_0)^3 \cdots + a_n(x - x_0)^n$,
where $a_i = \dfrac{f^{(i)}(x_0)}{i!}$

- The idea behind Taylor polynomials approximating a function $f(x)$ is to focus on how f behaves at *one point* x_0. We match not only the y-values at x_0, but also the slopes (the first derivative), the concavity (the second derivative), and however many more derivatives we choose – n is the number of derivatives we're choosing to match.

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- Based on a few examples, it seems likely that the higher n is, the better an approximation $P_n(x)$ gives.

- We want to find a polynomial of degree n, $P_n(x)$ so that $P_n(x)$ and its first n derivatives **all** match $f(x)$ and its first n derivatives at $x = 0$.

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- **Goal:** show that in order for the first n derivatives to match at the base point $x = 0$, the coefficients a_0 through a_n must be given by the formula $a_i = \dfrac{f^{(i)}(0)}{i!}$.

Let $f(x) = \ln(x)$ and let $P_5(x)$ be the 5th order Taylor polynomial for $f(x)$ at $x_0 = 1$.

1. Find $P_5(x)$
2. Verify your answer by graphing $P_5(x)$ and $f(x)$ on the same set of axes.
3. Use $P_5(x)$ to find an approximation for $\ln(1/2)$ and for $\ln(2)$. Based on the graphs, will these be larger or smaller than the actual value of $\ln(1/2)$ and $\ln(2)$? How good approximations does it look like they are?
4. By looking at the graph of the approximation error $|\ln(x) - P_5(x)|$, find an interval centered at 1 in which the approximation error is less than .01.