Example:

Let $f(x) = \cos(x)$.

- $P_{20}(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + \frac{x^{18}}{18!} - \frac{x^{20}}{20!}$,

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- We can use $P_{20}(0.1)$ to approximate $\cos(0.1)$ (for example).
- How close is $P_{20}(0.1)$ to the actual value of $\cos(0.1)$?

Maple says that the first 45 decimal places in each are

$$\begin{aligned} \cos(0.1) &= 0.995004165278025766095561987803870294838576225 \\ P_{30}(0.1) &= 0.995004165278025766095561987803870294838576314 \end{aligned}$$

So according to Maple, $|\cos(0.1) - P_{20}(0.1)| = 8.896631 \times 10^{-44}$.

Is this the exact error in using $P_{20}(0.1)$ to approximate $\cos(0.1)$?

- *NO!*

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- Maple is not giving cos(0.1) exactly, it's approximating it.

  Therefore, we can't know the error exactly.

  When our approximation is cruddy, we can assume that Maple's is so much better than ours that it's essentially exact. But when our approximation is this close to Maple's, who's to tell which one is better?

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  When our approximation is cruddy, we can assume that Maple's is so much better than ours that it's essentially exact. But when our approximation is this close to Maple's, who's to tell which one is better?
- Also, when you think about it, comparing to either Maple's graphs or Maple's values is pretty bogus, **since Maple's just approximating really really well using Taylor polynomials.**
- If we want to know how accurate our approximation is, we need to take some other route.

  Taylor's theorem gives us an error bound formula for using a Taylor polynomial to approximate a function at a point.

Example:

Let $f(x) = \cos(x)$. How close *is* $P_{20}(0.1)$ to the actual value of $\cos(0.1)$?

Recall that Maple says that the first 45 decimal places in each are

$$\begin{aligned} \cos(0.1) &= 0.995004165278025766095561987803870294838576225 \\ P_{30}(0.1) &= 0.995004165278025766095561987803870294838576314 \end{aligned}$$

So $|\cos(0.1) - P_{30}(0.1)| = 8.896631 \times 10^{-44}$.

Let $f(x) = \sin(x)$.

1. Find $P_6(x)$ for $f(x)$ based at $x_0 = \frac{\pi}{2}$.
2. Verify your result by graphing $P_6(x)$ and $f(x)$ on the same set of axes with $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.
3. Use your results to approximate $\sin(1)$.
4. Use Taylor's Theorem to find a bound on the error incurred by using $P_6(1)$ to approximate $\sin(1)$.
5. Find a value of n so that $P_n(1)$ approximates $\sin(1)$ accurate to within 10^{-10}.