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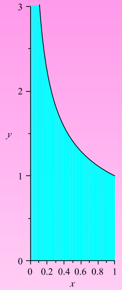

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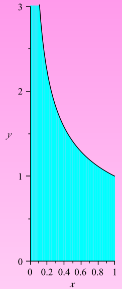

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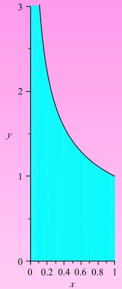

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$$2 = \int_0^1 \frac{1}{\sqrt{x}}\, dx = \int_0^1 1\, dy + \int_1^\infty \frac{1}{y^2}\, dy.$$

Definition:

If $f(x)$ is continuous on the interval $[a, \infty)$, we define the **improper integral** $\int_a^\infty f(x)\,dx$ to be

$$\int_a^\infty f(x)\,dx \stackrel{\text{def}}{=} \lim_{R\to\infty} \int_a^R f(x)\,dx.$$

Similarly, if $f(x)$ is continuous on the interval $(-\infty, a]$, we define

$$\int_{-\infty}^a f(x)\,dx \stackrel{\text{def}}{=} \lim_{R\to\infty} \int_{-R}^a f(x)\,dx.$$

In either case, if the limit exists (and equals some value L), we say that the improper integral **converges** (to L). If the limit does not exist (whether because it is infinite or for other reasons), we say that the improper integral **diverges**.

1. As $x \to \infty$, does each *integrand* diverge or converge (if so, to what?) Also, does each improper *integral* diverge or converge (if so, to what?)

a. $\int_1^\infty \frac{1}{x^3}\,dx$ b. $\int_1^\infty 1 + \frac{1}{x^2}\,dx$ c. $\int_1^\infty \frac{1}{x}\,dx$ d. $\int_0^\infty xe^{-x^2}\,dx$

2. Think about all the results you've seen, as well as the big picture.
   (a) Is it *necessary* that $f(x)$ converge to 0 as $x \to \infty$ in order for $\int_a^\infty f$ to converge to a finite number?

   (b) If $f(x)$ *does* converge to 0 as $x \to \infty$, *must* $\int_a^\infty f$ converge to a finite number? That is, is $f(x) \to 0$ a *sufficient* condition for $\int_a^\infty f$ to converge to a finite number?

Important Lessons:

1. There is a huge distinction between $f(x)$ converging – that is, $\lim_{x\to\infty} f(x)$ being finite – and $\int_a^\infty f(x)\,dx$ converging. Just because you can find $\lim_{x\to\infty} f(x)$, and it's a finite number, does **not** mean that $\int_a^\infty f(x)\,dx$ will be finite.

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2. In fact, if $\lim_{x \to \infty} f(x)$ exists **but is not 0**, $\int_a^\infty f$ diverges! No need to investigate any further.
3. If $\lim_{x \to \infty} f(x)$ **is** 0, $\int_a^\infty f$ may converge or it may diverge – to find out, you must actually do the antidifferentiation and the limit.