1. Translate the following calculations into summation notation. There is no need to actually find the value.
   1.1 The sum of the square roots of the first 100 integers

$$\text{sum} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{100} = \sum_{i=1}^{100} \sqrt{i}.$$

   1.2 The sum of the square roots of the first 10 even integers, beginning with 0

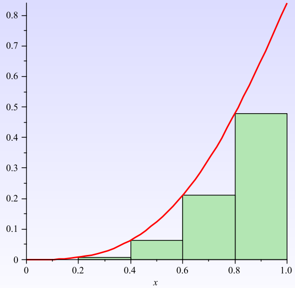
$$\begin{aligned} \text{sum} &= \sqrt{0} + \sqrt{2} + \sqrt{4} + \sqrt{6} + \sqrt{8} + \sqrt{10} + \sqrt{12} + \sqrt{14} + \sqrt{16} + \sqrt{18 \\ &= \sum_{j=0}^{9} \sqrt{2j} \\ &\textbf{or} \\ &= \sum_{k=1}^{10} \sqrt{2(k-1)} \end{aligned}$$

2. Let $I = \int_0^1 x \sin(x^2)\ dx$

2.1 Sketch L_5:

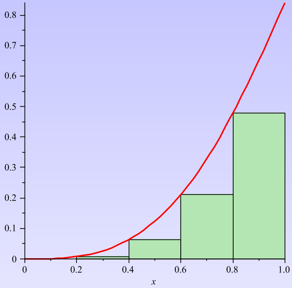
- 5 subintervals
- Each subint must have width $\Delta x = \frac{1-0}{5} = .2$.
- Partition: $0 < .2 < .4 < .6 < .8 < 1.0$.
- **Left** sum $\Rightarrow$ heights of rectangles determined by left endpoints of each subinterval
  – that is, by the heights at $x = 0$, $x = .2$, $x = .4$, $x = .6$, and $x = .8$.

Thus our left sum with 5 subintervals will look like:



2.2 Write out L_5 without Sigma Notation.

- $L_5 =$ the sum of the areas of these 5 rectangles.
- Base of each $= .2$
- Height of each is f(left endpoint).



$$
\begin{aligned}
L_5 &= .2 \cdot f(0) + .2 \cdot f(.2) + .2 \cdot f(.4) + .2 \cdot f(.6) + .2 \cdot f(.8) \\
&= .2 \cdot 0 + .2 \cdot (.2 \sin(.04)) + .2 \cdot (.4 \sin(.16)) + .2 \cdot (.6 \sin(.36)) + .2 \\
&= .2(0 + .2 \sin(.04) + .4 \sin(.16) + .6 \sin(.36) + .8 \sin(.64))
\end{aligned}
$$

2.3 Use Sigma notation to write L_5.

- The patterns in L_5 are easiest to see in the first line of 2.2.

$$L_5 = .2 \cdot f(0) + .2 \cdot f(.2) + .2 \cdot f(.4) + .2 \cdot f(.6) + .2 \cdot f(.8)$$

- Even easier to see if we work with fractions!

$$L_5 = \frac{1}{5}\left(f(0) + f\left(\frac{1}{5}\right) + f\left(\frac{2}{5}\right) + f\left(\frac{3}{5}\right) + f\left(\frac{4}{5}\right)\right).$$

- Each term inside the parentheses is the same **except** the numerators in the fractions are consecutive numbers from 0 to 4.
- We're adding up the term $f\left(\frac{i}{5}\right)$ over and over again as i goes from 0 to 4.

Thus in sigma notation, we have:

$$L_5 = \frac{1}{5}\sum_{i=0}^{4} f\left(\frac{i}{5}\right).$$

- Sigma notation is just shorthand for a sum.
- Many different ways to write the same sum.
- Rules that apply to sums still apply: $L_5 = \sum_{i=0}^{4} \frac{1}{5} f\left(\frac{i}{5}\right)$.

2.4 Calculate the numerical value of L_5. Without finding the exact value of I, decide whether L_5 over-estimates or under-estimates I.

$$\begin{aligned} L_5 &= .2(0 + .2\sin(.04) + .4\sin(.16) + .6\sin(.36) + .8\sin(.64)) \\ &= 0.1521692085. \end{aligned}$$

Because the height of each rectangle is at or below the height of the function over the same subinterval, the area of each rectangle is less than the area under the function over the subinterval, and so the left sum is an *under-estimate* for I.

2.5 Write L_{10} and L_{50} using sigma notation. Do these over- or under-estimate I?

$$L_{10} = \sum_{i=0}^{9} \frac{1}{10} f\left(\frac{i}{10}\right) \qquad L_{50} = \sum_{i=0}^{49} \frac{1}{50} f\left(\frac{i}{50}\right).$$

Because $x\sin(x^2)$ is increasing on the interval $[0, 1]$, the left-hand endpoint of each rectangle (no matter how narrow or wide) will always be the lowest point of the curve over the subinterval, and so a rectangle with that height will have smaller area than the curve over that subinterval. Thus both L_{10} and L_{50} will under-estimate I.