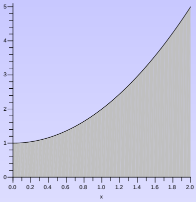
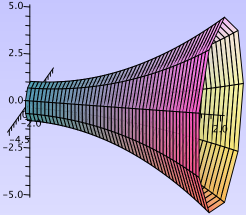
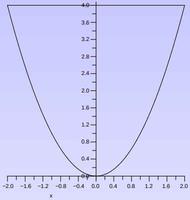
1. The solid formed when the graph of $y = x^2 + 1$ from $x = 0$ to $x = 2$ is rotated about the x-axis.

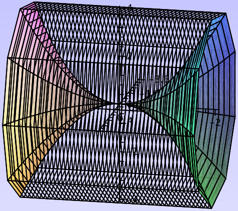




$$\begin{aligned}
\text{Volume} &= \int_0^2 A(x)\,dx = \pi \int_0^2 \text{radius}^2\,dx = \pi \int_0^2 (x^2+1)^2\,dx \\
&= \pi \int_0^2 x^4 + 2x^2 + 1\,dx = \pi\left(\frac{x^5}{5} + \frac{2x^3}{3} + x\right)\Big|_0^2 \\
&= \pi\left[\left(\frac{32}{5} + \frac{16}{3} + 2\right) - 0\right] = \pi\left(\frac{96+80+30}{15}\right) = \frac{206\pi}{15}
\end{aligned}$$

2. The solid formed when the region bounded by $y = x^2$ and $y = 4$ is rotated about the x-axis.

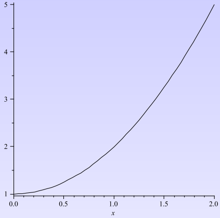


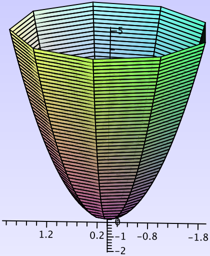


$$\begin{aligned}
\text{Volume} &= 2 \cdot \int_0^2 A(x)\,dx = 2\pi \int_0^2 (\text{outer } r)^2 - (\text{inner r})^2\,dx \\
&= 2\pi \int_0^2 (4)^2 - (x^2)^2\,dx = 2\pi \int_0^2 16 - x^4\,dx \\
&= 2\pi\left(16x - \frac{x^5}{5}\right)\Big|_0^2 = 2\pi\left[\left(32 - \frac{32}{5}\right) - 0\right] = 2\pi\left(\frac{128}{5}\right) = \frac{256\pi}{5}
\end{aligned}$$

3(a). $y = x^2 + 1$ from $x = 0$ to $x = 2$, rotated about the y axis.

Because we were only told to rotate the curve $y = x^2 + 1$ and find the enclosed volume, notice that even though we're rotating the same curve as we did in (1), we're enclosing a totally different volume:





Rotating around y-axis $\Rightarrow$ Circular cross-section are perp to y-axis
$\Rightarrow$ Integrate with respect to y, and radius is $x = g(y)$.
Rewriting $y = x^2 + 1$ as $x = g(y)$: $x = \sqrt{y-1}$.
Lowest y: $y = 1$; Biggest y: $y = 5$.

3(a) (continued)

$$\begin{aligned}
\text{Volume} &= \int_1^5 A(y)\,dy = \pi \int_1^5 \text{radius}^2\,dy \\
&= \pi \int_1^5 (\sqrt{y-1})^2\,dy \\
&= \pi \int_1^5 y - 1\,dy \\
&= \pi \left(\frac{y^2}{2} - y \right) \Big|_1^5 \\
&= \pi \left[\left(\frac{25}{2} - 5 \right) - \left(\frac{1}{2} - 1 \right) \right] \\
&= 8\pi
\end{aligned}$$

3(b) Rotate first quadrant portion of region bdd by $y = x^2$ and $y = 4$ about y-axis.

If you rotate the *whole* region, only need to rotate by 180° to form a solid with circular cross-sections. Rotating by 360° would overlap what you've already formed – you'd double the volume. *Also*, it's harder, because you'd have to do two functions. So ... you'd work harder to get the wrong answer.

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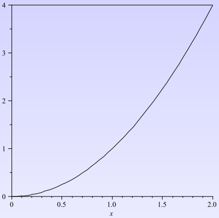
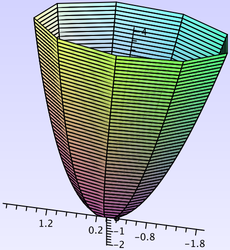


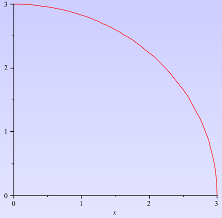


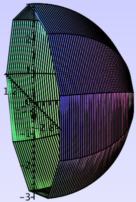
Circular cross-sections are perp to y-axis $\Rightarrow r = x = g(y) = \sqrt{y}$, from $y = 0$ to $y = 4$.

3(b) (continued)

$$\begin{aligned}
\text{Volume} &= \int_0^4 A(y)\,dy \\
&= \pi \int_0^4 \text{radius}^2\,dy \\
&= \pi \int_0^4 (\sqrt{y})^2\,dy \\
&= \pi \int_0^4 y\,dy \\
&= \pi\left(\frac{y^2}{2}\right) \text{ from 0 to 4} \\
&= \pi\left[\left(\frac{16}{2}\right) - (0)\right] \\
&= \pi(8)
\end{aligned}$$

4. The sphere of radius r.
Form sphere of radius r by rotating upper half of a circle of radius r about x-axis. In fact, just rotate upper right quarter-circle, and multiply by 2.





A circle of radius r, centered at the origin, has equation $x^2 + y^2 = r^2$, so the upper half has equation $y = \sqrt{r^2 - x^2}$.

4. (continued)

$$
\begin{aligned}
V &= 2\left[\pi \int_0^r (\sqrt{r^2 - x^2})^2 \; dx\right] \\
&= 2\pi \int_0^r r^2 - x^2 \; dx \\
&= 2\pi r^2 x - \frac{x^3}{3}\bigg|_0^r \\
&= 2\pi[(r^3 - \frac{r^3}{3}) - (0)] \\
&= \frac{4\pi r^3}{3}
\end{aligned}
$$