1. Differentiate the following functions by hand.

   1.1 $f(x) = \ln(x^2 + 3)$
   chain rule!

   $$f'(x) = \frac{1}{x^2 + 3} \cdot 2x$$

   1.2 $g(w) = w \cos(e^w)$
   product rule and chain rule!

   $$g'(w) = w \cdot -\sin(e^w) \cdot e^w + \cos(e^w)$$

2. Find two antiderivative for $p(x) = 3x^5 + 7x^4 - \frac{3}{x} + \frac{11}{x^2}$ by hand.

One antiderivative is

$$P(x) = \frac{3}{6}x^6 + \frac{7}{5}x^5 - 3\ln|x| - \frac{11}{x}$$

Notice: I didn't put in a **+ C** because that refers to the family of *all* antiderivatives.

All antiderivatives of $p(x)$ differ by at most a constant, *and* adding any constant to $P(x)$ will give me a second antiderivative.

For example, a second antiderivative is

$$Q(x) = \frac{3}{6}x^6 + \frac{7}{5}x^5 - \frac{1}{3} \cdot \frac{1}{3}x^3 + 11x + e.$$

3 Find the function $v(t)$ satisfying the conditions that $v'(t) = 2e^t - 3\cos(3t)$ and $v(0) = -3$.

- If you've learned and remember a technique for this, it's okay to use it.
- If not, just use the guess and check method: guess an answer, then check it by differentiating.

The family of antiderivatives is

$$v(t) = 2e^t - \sin(3t) + C$$

I need the specific antiderivative that satisfies $v(0) = -3$, so

$$-3 = v(0) = 2e^0 - \sin(0) + C = 2 + C \Longrightarrow C = -5.$$

Thus $v(t) = 2e^t - \sin(3t) - 5$.

4. Suppose that $f(x) = x^2 - 3e^{x^2} + 4$, and $A_f(x) = \int_{-5}^{x} f(t)\, dt$.

4.1 Is the graph of $f(x)$ increasing or decreasing at $x = -2$?

$$f'(x) = 2x - 6xe^{x^2} \Rightarrow f'(-2) = -4 + 12e^4 > 0.$$

f is increasing at $x = -2$.

4.2 Is the graph of $f(x)$ concave up or concave down at $x = -2$?

$$f''(x) = 2 - 12x^2e^{x^2} - 6e^{x^2} \Rightarrow f''(-2) = 2 - (48 + 6)e^4 < 0.$$

f is concave down at $x = -2$.

4.3 Is the graph of $A_f(x)$ increasing or decreasing at $x = -2$?
Since A_f is an antiderivative of f, A_f' is f.

$$A_f'(x) = f(x) = x^2 - 3e^{x^2} + 4 \Rightarrow A_f'(-2) = 4 - 3e^4 + 4 < 0.$$

A_f is decreasing at $x = -2$.

4.4 Is the graph of $A_f(x)$ concave up or concave down at $x = -2$?
Since $A_f''(x) = f'(x)$,

$$A_f''(-2) = f'(-2) = -4 + 12e^4 > 0 \Rightarrow A_f \text{ concave up at } x = -2$$

5. Find the signed areas given by the following integrals:

5.1 $\int_1^4 \pi - x^{-3/2}\, dx$

Use FTC, Part 1:

$$\begin{aligned} \int_1^4 \pi - x^{-3/2}\, dx &= \left[\pi x - \frac{x^{-1/2}}{-1/2}\right]_1^4 = \left[\pi x + \frac{2}{\sqrt{x}}\right]_1^4 \\ &= (4\pi + 1) - (\pi + 2) = 3\pi - 1 \end{aligned}$$

5.2 $\int_2^3 6z^5 + \frac{5}{z^{10}}\, dz$

$$\begin{aligned} \int_2^3 6z^5 + \frac{5}{z^{10}}\, dz &= \left[z^6 - \frac{5}{9}z^{-9}\right]_2^3 = \left(3^6 - \frac{5}{9 \cdot 3^9}\right) - \left(2^6 - \frac{5}{9 \cdot 2^9}\right) \\ &\approx 949.44 \end{aligned}$$