Jill Grissom
33 Kumquat Ave, Apt 3B
Coral Gables, FL 33124
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Math 104 Students
Wheaton College
Norton, MA 02766

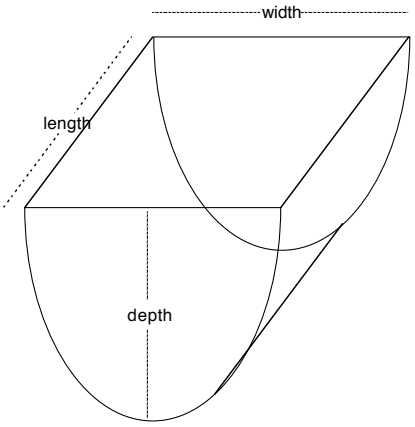
Dear Calculus Students:

I've always wanted to be a Crime Scene Investigator like my Uncle Gil used to be, and have been lucky enough to get an internship with the local Crime Scene Investigative Unit here in Miami, where I've started college. I've been given a great chance to contribute to an investigation, but I've reached a point where I'm stuck. I think the problem is that I don't have enough mathematical background yet – I've just started Calc 1 – so I'm turning to you for help.

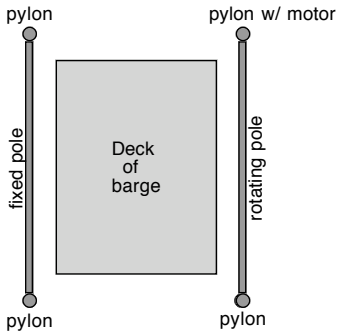
Here's the deal. Some off-duty police officers headed off toward their favorite uninhabited island for a day of scuba diving. When they got to the island's lagoon, they noticed four pylons jutting up above the water's surface that hadn't been there before. Curious, they dove below the surface and found a sunken barge lying on the lagoon floor, neatly lined up with the pylons. Investigating further, they discovered that the boat was in remarkably good condition for a sunken ship – if it weren't for the fact that it was sunk, you'd think it was perfectly sea-worthy. Their conclusion: for some reason, someone wanted to hide the ship for a while and had decided that sinking it was the best way ... but apparently had gone to a lot of effort not to damage the ship any more than necessary during the sinking process (perhaps they hope to raise it and use it later?).

Since hiding a ship by sinking it counts as polluting the ocean waters (and because it indicates some other larger crime involving the boat), the police officers reported what they'd found. Here's a summary:

- Looking at the barge from the front or back, the barge is seemingly parabolic in shape, while the deck itself is rectangular. Here's a rough sketch of the front view:




- The four pylons are all about the same height and they form a rectangle whose sides are parallel to the sides of the barge's deck. Two sides of this rectangle have been attached by poles, and the barge is centered between these poles. One of those poles is fixed, the other seems to be able to turn easily on its axis (like an axle)– and it looks as if a motor has been attached to one end of that turning pole. Here's another sketch, this time from the top view:




- trapped under the barge were some scraps of material that the scuba-diving police officers brought back to the lab. Upon investigation, one of the lab techs learned that this material is super-strong, non-stretchy, and really flexible.

I suggested to my boss that perhaps the boat was was slowly lowered to the ocean floor in a sling made out of this super-strong, non-stretchy, really flexible material, and that's why the boat was barely damaged. More

specifically, I proposed that the people who sunk the boat could have had a huge roll of that odd non-stretchy material wound on the pole that turns – kind of like a roll of paper towel. Then they could have steered the barge between the poles, unrolled some of the material, passed it under the barge, and attached it to the fixed pole on the other side, forming the sling. Next, they could have pumped the bilge tanks full, so that the boat would sink. Rather than sinking to the bottom of the lagoon in an uncontrolled way, however, it would settle into the sling. Then the motor attached to the rotating axle-like pole would control the turning of the axle so that the material would slowly unroll, allowing the boat to sink at a controlled speed until it gently touches the bottom. Then they could have gathered up the material and gone on their merry way. If that's the case, then the people who sunk the boat would have needed **a lot** of that odd material, and perhaps we could trace the criminals that way.

My boss told me that since it's my idea, I can pursue it! This may be my big chance, but unfortunately, as I mentioned earlier, I've reached a point where I'm stuck. Let me tell you what I've done so far, and where I'm trying to go.

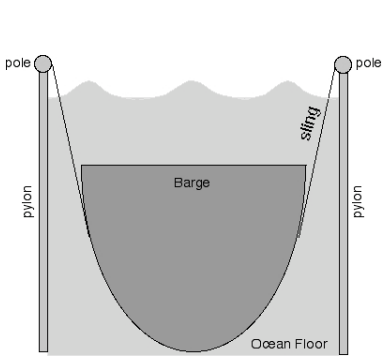
- The most obvious thing to do was to find out if any very large orders of the material we found under the barge had been placed recently. Unfortunately, when we contacted the manufacturers, we learned they've gotten more large orders than we care to trace, so we need to narrow it down some more. If we can get a pretty close estimate of how much material the criminals would have needed to build the sling I'm proposing, we can contact the manufacturers again and ask them who ordered around that much of the fabric. That should help winnow the field of suspects down quite a bit.

  It's in trying to calculate the length of fabric the criminals would have needed that I get stuck, and need your help.

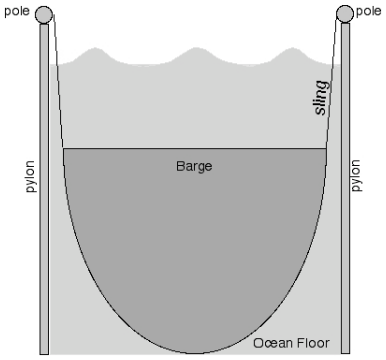
- Before diving into calculations, I wanted to make sure that my idea for gently sinking the ship in a sling would work, so I tested it by making several different models. I found that indeed my idea does work. To replicate what the police officers found, I *did* have to rig my barge-models so that the flat deck stayed flat throughout the sinking process and so that it would sink straight down with no drifting side-to-side ... but if I can do it, so could the criminals.

- I don't know whether it will turn out to be relevant, but as I experimented with my models, I found something interesting about the sling:

how the sling fits to the side of the boat differs depending on the height and placement of the pylons compared to the depth and width of the barge. In some models, the sling clings to side of the barge part of the way up the sides, but then breaks away from the sides of the barge and heads straight up to the poles, as in the sketch on the left on the next page. In others, the sling clings to the side of the barge all of the way to the "top" of the barge, and only then heads straight over to the poles, as in the sketch on the right.




Sling leaves sides of barge before reaching top




Sling hugs sides of barge all the way to the top

- In order to proceed any further - including figuring out which of these scenarios would have occurred, I needed measurements. Two Miami CSI volunteered to go to the lagoon on their off day. Unfortunately the seas were very choppy that day, and despite their best efforts, they couldn't get consistent results. In the end, they presented me with two radically different sets of measurements, indicating the extent of the roughness of any estimate we'll be able to come up with. I've summarized the two sets of measurements below.

| | barge width | barge length | barge depth | pylon height | ht of pylon above water | pole lengths | pole width |
|---|---|---|---|---|---|---|---|
| Set 1 | 24 ft | 40 ft | 32 ft | 46 ft | 3 ft | 51 ft | 32 ft |
| Set 2 | 23.5 ft | 42 ft | 31 ft | 47 ft | 2 ft | 50 ft | 28 ft |

- By this time, I was a pro at making models, so I made a scale model of Set 1 of the measurements. For that model, the sling left the side of the barge before reaching the top, as in the sketch on the left. I

noticed, though, that it left the side of the barge pretty close to the top, so if the measurements were off by just a little bit (as we know they very likely are), the actual barge could have been in the situation on the right.

- I tried just measuring the fabric on my attempt at a scale model and then adjusting proportionally, but it didn't work too well – the only fabric I have stretches too much, and the budget doesn't allow for interns like me to go out and buy high-tech fabric samples. I called the fabric manufacturers with my less-than-fabulous estimate, and it turns out that many orders around that size had been placed – I need some way to calculate more closely how much fabric would have been needed.

- And that's where I got stuck – I have no idea where to start calculating the length of fabric needed if the first set of measurements are accurate.

- At that point, I didn't want to bother making a scale model of the second set of measurements in case it turn out that there's just no way of doing this.

In short, I've got a plan, and I've made models, and now I'm at a loss. So I have several questions for you:

- Based on the measurements in Set 1, how long a roll of material would the criminals have needed? Could you explain in enough detail so that hopefully, if we ever get a more exact set of measurements and it turns out the actual barge/pole scenario falls into the situation where the sling leaves the side of the barge before reaching the top?

- If the second set of measurements is correct, is it still possible that we're in that same scenario, where the sling leaves the sides of the barge before the top? If so, given how far apart these two sets of measurements are, the chances are reasonably good that this scenario is what actually happened, and we can focus our attention on that. In that case, how long a roll of fabric would be needed?

- If the second set of measurements is correct but in that case it turns out the sling doesn't leave the top of the barge before the top, first ... could tell me how you can tell? Also, of course, how much fabric would be needed?

- Perhaps I'm totally nuts, but I just have a feeling that whenever we get exact measurements for the width, depth, and length of the boat, the height of the pylons as well as how far apart they are, the length of the poles, and how far up the pylons the water goes, the situation on the right is going to be the one we're in. Could you work out a general formula for how long a roll of material the criminals would need, based on all these various dimensions involved, when we're in the situation on the right?

- *If* you have time, I'd appreciate it if you'd investigate the differences between the above two situations in more detail. I'm really curious about what's going on: what are the critical differences between the situation on the left and the one on the right? It seems to have something to do with how far apart and how tall the pylons are, compared to how wide and tall the barge is – what is the "dividing line" where the situations switch?

I need a response on or before October 9, so that I have time to read through and be sure I understand it. I'd appreciate it if you'd not merely send specific answers to the above questions, but also include explanations of what you did, so that I can follow, understand, and be sure that everything makes sense to me. If I'm right that answering these questions does end up using Calculus, I should probably tell you that we've just begun differentiation, so that's all I know. I should probably also tell you that I might well share your results with the senior CSIs.

Sincerely yours,
Jill Grissom