Some Sample Problems for Midterm 2

Midterm 2 will cover Sections 3.2, 5.2, 5.4, 6.1, 6.2, 6.6, and 8.1 from your textbook, as well as 9.1 and 9.2 from the textbook sections I put on-line. It will also, of course, implicitly cover most of the previous material – in order to integrate, you must know sections 2.8, and chapter 4; I may choose to test your understanding of integration by parts or improper integrals through an area problem, or your understanding of volume through a u-substitution.

As with the last exam, you may have one sheet of notes. The format is the same as before: handwritten (by you) notes on one-side of a standard 8 1/2 x 11 sheet of paper.

Also as before, you may begin the exam at 12:30pm, but may not continue taking the exam beyond 1:55. (If you have class up until 1, talk to me or e-mail me).

The usual words of caution: The following problems neither are a practice test nor should replace other studying. The test may not cover exactly the same material as these problems cover, and the test problems will not be just like these. The idea is simply to give you additional problems to practice on. *The more problems you do, the better!*

You are responsible for all material covered in your reading, whether or not we covered it in class.

- As always, you should be able to differentiate quickly and accurately. Remember not only the derivatives of "elementary" functions such as the trig and inverse trig functions, e^x and $\ln(x)$ but also the product, quotient and chain rule.
- Be able to recognize which antidifferentiation method is called for, and to antidifferentiate quickly and accurately using either.
- Don't count on rote antidifferentiation problems – that's what the antidifferentiation exam is for. Be prepared for more theoretical questions related to §6.2, or for antidifferentiation to be just one small part of various larger problems.

  Practice antidifferentiation no matter when you passed the antidiff exam.
- You should remember how to find the area of a region bounded by various curves. This includes finding where two curves intersect algebraically.
- Be able to set up arclength problems and evaluate them when possible.
- Be comfortable doing volume of a solid of revolution whether it's rotated about the x- or y- axis , or some other horizontal or vertical line. You should of course be able to do a solid whether it's cross-sections look like solid discs or washers.
- Understand what distinguishes an improper integral from a definite integral.

- Understand what it means for an improper integral to converge, and to diverge. What is the difference (conceptually) between the *integrand* converging and the *integral* converging? Can an improper integral diverge even if the integrand converges?
- Can $\int_1^\infty f(x)dx$ converge if $\lim_{x\to\infty} f(x) = .01$?
- Have a "lexicon" in your head of standard examples of improper integrals whose convergence or divergence we know. This lexicon should include not only $\frac{1}{x^p}$ for various values of p and over various intervals –but also e^{-x} over $[a, \infty]$.
- Understand when comparisons to determine convergence or divergence of an improper integral are useful and when they are not.
- Know that you **always** have to convince the reader (whether in a class situation or real life) that a comparison you've chosen to make is true, either algebraically or graphically.
- Know the idea behind Taylor polynomials –how do they approximate functions? do they approximate them everywhere?– and be able to find nth degree Taylor polynomial of a function, centered at $x = 0$ or at any other point.
- What is the connection between the nth degree Taylor polynomial for a function centered at x_0, and any lower order Taylor polynomial for that function centered at that same point?
- Understand and know how to use Taylor's Theorem.
- Understand l'Hopital's rule, and be able to recognize when a need for it arises in the middle of a problem, for instance in the midst of evaluating an improper integral.
- Understand what sequences are, what convergence of a sequence means, and when we know that a sequence must converge. Also know what the terms of a sequence are, including what we mean by *general term*.
- Be able to do all the assigned problems, and all the following problems.

1. Use arclength to compute the length of the line $y = mx + b$ from $x = A$ to $x = B$. Does the answer agree with the usual distance formula?
2. Sketch the region bounded by the graphs $y = \sqrt{8x}$ and $y = x^2$.

(a) Find the volume of the solid formed when the region is rotated about the x-axis.

(b) Find the volume of the solid formed when the region is rotated about the y-axis.

(c) Find the volume of the solid formed when the region is rotated about the line $y = -1$.

3. A drinking glass has circular cross sections. The glass has height 5 inches, bottom diamter 2 inches, and top diameter 3 inches. How much liquid can the glass hold?

4. Let $I = \int_0^{2\pi} \sqrt{1+\cos(x)^2}\, dx$.

   (a) Interpret I as giving the area of a certain region R. What is R? Sketch or plot R.

   (b) Interpret I as giving the length of a curve C. What is C? Sketch C.

   (c) Interpret I as giving the volume formed when a function $f(x)$ is rotated about the x-axis. What is $f(x)$? Plot $f(x)$.

5. **Gabriel's Trumpet** Consider the graph of $f(x) = \frac{1}{x}$ for $x \geq 1$.

   (a) Find the area below the graph and above the x-axis.

   (b) Find the volume of the solid formed when the graph is rotated about the x-axis.

6. Find the exact value of $\int_1^{\infty} \frac{1}{1+x^2}\, dx$.

7. Suppose that f is continuous everywhere and that $\int_2^{\infty} f(x)\, dx$ converges. Is the statement that therefore $\int_5^{\infty} f(x)\, dx$ also converges: always true, sometimes true, or never true?

8. Do the following integrals converge or diverge? You do not need to find the values of the convergent integrals.

   (a) $\int_2^{\infty} \frac{x}{x^2-2}\, dx$ (c) $\int_0^{12} \frac{1}{\sqrt[3]{x}+x}\, dx$

   (b) $\int_8^{\infty} \frac{1}{x+e^x}\, dx$ (d) $\int_1^{\infty} \frac{x^2+1}{x^2}\, dx$

9. (a) Find the seventh order Taylor polynomial for $f(x) = e^x$ at $x_0 = 0$.

(b) How close will $P_7(x)$ approximate $e^{.5}$?

(c) Use your result for part (a) to find the 14th order Taylor polynomial for $g(x) = e^{x^2}$ based at $x_0 = 0$.

(d) Use your result for part (c) to approximate $\int_0^1 e^{x^2}\,dx$.

10. Find a symbolic expression for the general term a_k of the sequence

$$\frac{1}{2}, -\frac{1}{7}, \frac{1}{28}, -\frac{1}{63}, \frac{1}{126}, \ldots$$

Hint: Notice the similarity between the denominators and the sequence $\{1, 8, 27, 64, 125, \ldots\}$.

11. Determine whether the following sequences converge or diverge. If they converge, what do they converge to?

(a) $a_k = \dfrac{\sin(e^{k^5})}{e^{k^2}}$

(b) $b_j = \dfrac{14j^2 - 1}{2j^2 + 1000j + 256}$

(c) $c_i = \dfrac{i!1.0001^i}{(i+1)!}$

(d) $d_n = e^{n/100} \ln(50000n)$