Some Sample Problems for the Final

The final will be given Tuesday 12/15 from 9am - 12pm in **Knapton 214**. *Assume it will take the full 3 hours.*

The final will be comprehensive. That is, it will cover Sections 2.8, 3.2, 4.1-4.7, 5.1, 5.2, 5.4, 6.1, 6.2, 6.5, 6.6, Taylor Polynomials, 8.1-8.8. I know it seems like this is a lot of material (and it is), but a lot of the later material builds upon the earlier material, so I think you'll find that a lot of the early sections won't take that much review because you already know it. Anything we've learned is fair game, but it might help if you keep in mind that my main goals with the final is to check whether you've learned and understood all the most important topics, and also to find how much you've been able to put together and connect the various topics. That means that I may not ask questions that focus on small finicky details (although your definition of small finicky detail and mine may differ), but that I may ask questions that combine ideas that you haven't seen combined before.

I will continue to allow you to have one sheet of notes. The format is the same as it has been for the midterms – handwritten (by you) notes on one-side of a standard 8 1/2 x 11 sheet of paper. I know that this means more material on the same size piece of paper and this is quite intentional – I want you to really think about what you need on there.

This study guide only covers material covered since Midterm 3. To review the older material, redo the previous studyguides (available on my web site) and do problems from the text and from the review sections in the text.

I have thrown these questions together quickly, so make no guarantee that they formulate a comprehensive review of the newest material. The test may not cover exactly the same material as these problems cover, and the test problems will not be just like these.

As always, the more problems you do, the better!

You are responsible for all material covered in your reading, whether or not we covered it in class.

- Know the general form of a power series.
- Know how power series differ from the series we covered previously.
- Understand why we study power series.
- Know how to find the interval of convergence of a power series, including endpoint behavior, and understand what the radius of convergence is. Really grasp that on that interval of convergence, a power series *is* a function.

- Know how to differentiate and integrate power series term-by-term. Know that the radius of convergence remains unchanged, although the endpoint behavior may change. Understand that we can not be sure that these differentiation and integration techniques apply to series that *are* functions but that are *not* power series.

- Also know how to go from a known power series representation of a function to a new function by substituting new values in for x. (For instance, we can use the power series representation for $\frac{1}{1-x}$ to find the power series representation for $\frac{1}{1+x^2}$.)

- Given a function $f(x)$, know how to form the Taylor series for f based at any base point x_0.

- Be aware that Taylor series are a specific type of power series, and as such, we find the interval of convergence (including endpoint behavior) the same way.

- Know how to use the new version of Taylor's Theorem to find whether the Taylor series for $f(x)$ converges to the function $f(x)$ on its interval of convergence.

- (This is actually old material) - Know how to use Taylor series/Taylor polynomials to approximate specific values of $f(x)$, for instance how to use the Taylor series for $\sin(x)$ to approximate $\sin(1)$.

- Know how to find a new Taylor series from an old one – for instance, if you know the Taylor series for $\sin(x)$, how can you find the Taylor series for $\int \sin(x^2)\,dx$.

- Similarly, now how to use Taylor series to calculate a definite integral.

- Be able to do all the assigned problems, and all those below

1. Determine the radius and interval of convergence of the following power series.

   (a) $\displaystyle\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{(j+2)5^j}(x+4)^j$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^3}(x-7)^n$

2. Determine the interval of convergence of the power series $\sum_{k=0}^{\infty}(4x-2)^k$, and find the function to which this power series converges on its interval of convergence.

3. (a) Find a power series representation of $f(x) = \dfrac{3}{2-8x^2}$, based on what you know about geometric series. Also find the interval of convergence for this power series.

   (b) Use your results to (a) to find a power series representation for $\dfrac{48x}{(2-4x^2)^2}$. What is the radius of convergence of this power series?

4. Let $f(x) = e^{3x}$.

   (a) Find the Taylor series for $f(x)$ about $c = 1/3$, (from scratch).

   (b) Find the interval of convergence for this Taylor series.

   (c) Show that this Taylor series does in fact converge to $f(x)$.

5. Use the appropriate known Taylor series to find the Taylor series for $\int x^2 e^{-x^2}\, dx$ about $x_0 = 0$.

6. Use the appropriate known Taylor series to approximate $e^{0.4}$ accurate to within 10^{-11}.

7. The weight (force due to gravity) of an object of mass m and altitude x miles above the surface of the earth is $w(x) = \dfrac{mgR^2}{(R+x)^2}$, where R is the radius of the earth and g is the acceleration due to gravity.

   (a) Show that $w(x) \approx mg\left(1 - \dfrac{2x}{R}\right)$.

   (b) Estimate how large x would need to be to reduce the weight by 10% (as compared to what it is on the surface of the earth).