In Exercises 1 and 2, suppose that $a(x)$ is continuous, positive, and decreasing for all $x \geq 1$ and that $a_k = a(k)$ for all integers $k \geq 1$.

1. Rank the values $\int_n^\infty a(x)\, dx$, $\sum_{k=n+1}^{\infty} a_k$, and $\int_{n+1}^\infty a(x)\, dx$ in increasing order. *Hint:* Draw a carefully annotated picture

2. Draw a carefully annotated picture that shows that if $a(x)$ is continuous, non-negative, and decreasing:

   (a) $\int_1^{n+1} a(x)\, dx \leq \sum_{k=1}^{n} a_k$

   (b) $\sum_{k=2}^{n} a_k \leq \int_1^n a(x)\, dx$

   (c) $\sum_{k=n+1}^{\infty} a_k \leq a_{n+1} + \int_{n+1}^\infty a(x)\, dx$

In Exercises 3 and 4, use the integral test to find upper and lower bounds on the limit of the series.

3. $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$

4. $\sum_{k=1}^{\infty} je^{-j}$

In Exercises 5 and 6, determine whether the series converges or diverges. If the series converges, find a number N such that the partial sum S_N approximates the sum of the series within 0.001. *If the series diverges, find a number N such that $S_N \geq 1000$.*

5. $\sum_{n=1}^{\infty} \frac{\arctan(n)}{1+n^2}$

6. $\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$