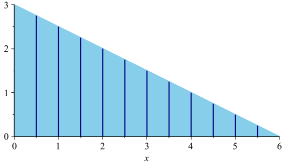
Daily WeBWorK, problem 2

Calculate the volume of the solid under the graph of the function $f(x, y) = xy$ over the triangular region $x + 2y \leq 6$, $x \geq 0$, $y \geq 0$.

$$\text{Signed Volume} = \iint_R xy\ dA.$$

Need to understand the region triangular region R to rewrite as an iterated integral.

$x + 2y \leq 6 \Rightarrow y \leq -\frac{x}{2} + 3 \Rightarrow R$ consists of region below $y = -\frac{x}{2} + 3$, above $y = 0$, to the right of $x = 0$:



Leftmost-x-value: $x = 0$
Rightmost-x-value: $x = 6$

$$\Rightarrow \iint_R xy\ dA = \int_0^6 \int_C^D xy\ dy\ dx$$

As x goes from 0 to 6:
bottom-most curve: $y = 0$
topmost-curve: $y = -\frac{x}{2} + 3$

$$\Rightarrow \iint_R xy\ dA = \int_0^6 \int_0^{3-x/2} xy\ dy\ dx$$

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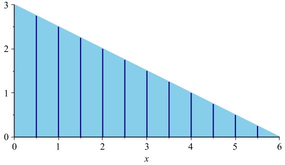
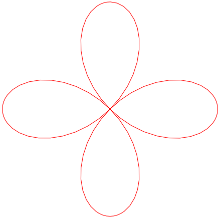
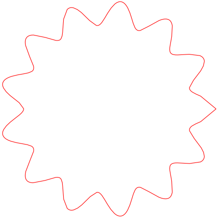
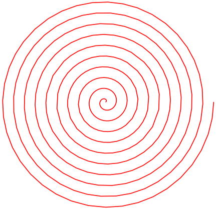


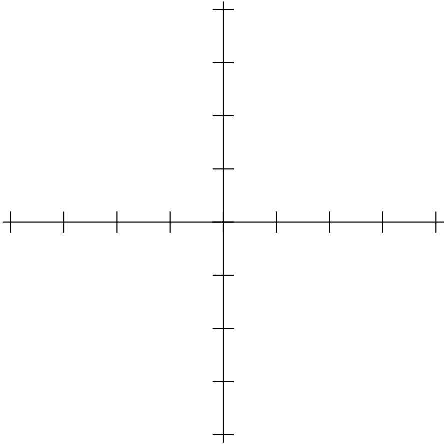
$$\begin{aligned} V &= \int_0^6 \int_0^{3-x/2} xy \; dy \; dx \\ &= \int_0^6 \left(x \cdot \frac{y^2}{2} \right) \Big|_0^{3-x/2} dx \\ &= \int_0^6 \left(x \cdot \frac{(3 - \frac{x}{2})^2}{2} \right) - 0 \; dx \\ &= \frac{1}{2} \int_0^6 9x - 3x^2 + \frac{x^3}{4} \; dx \end{aligned}$$

With polar coordinates, some new graphs are a snap!

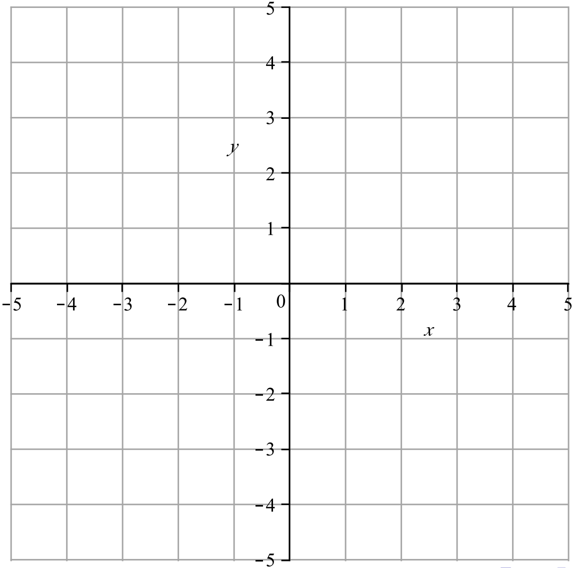
  

Each of these is just the graph of a single simple function.

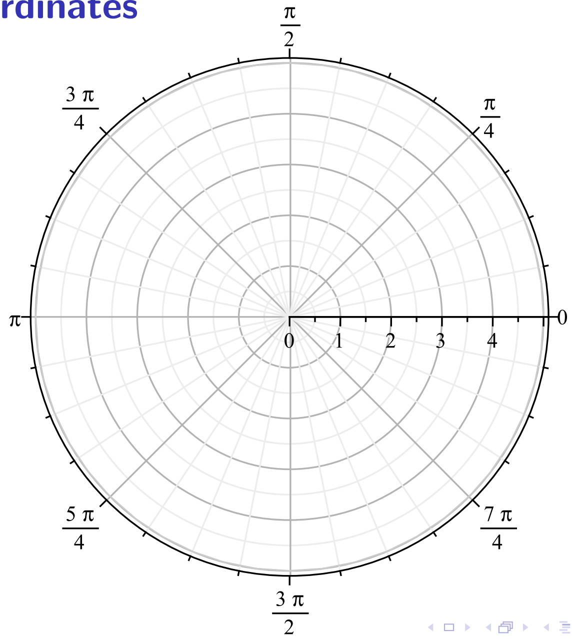
Identifying Points in the Plane



Rectangular Coordinates



Polar Coordinates



In Class Work

1. Plot the point $\left(-2, 4\pi/3\right)$ in polar coordinates; verify by converting to rectangular coordinates.

2. Convert the rectangular point $(-3, -5)$ to polar coordinates.

3. Sketch the graph of $\theta = 3\pi/4$ (by plotting points)

4. Sketch the graph of $r = 4$ (by plotting points)

5. Sketch the graph of $r = \sin(4\theta)$ (by plotting points)

Solutions

1. Plot the point $\big(-2, 4\pi/3\big)$ in polar coordinates; verify by converting to rectangular coordinates.
I will just show verifying:

$$x = r\cos(\theta) = -2\cos\left(\frac{4\pi}{3}\right) = -2 \cdot -\frac{1}{2} = 1$$

$$y = r\sin(\theta) = -2\sin\left(\frac{4\pi}{3}\right) = -2 \cdot -\frac{\sqrt{3}}{2} = \sqrt{3}$$

Thus the polar point $\big(-2, 4\pi/3\big)$ is equivalent to the rectangular point $\big(1, \sqrt{(}3)\big)$.

Solutions

2. Convert the rectangular point $(-3, -5)$ to polar coordinates.

$$r = \sqrt{x^2 + y^2} = \sqrt{9 + 25} = \sqrt{34}$$

$$\tan(\theta) = \frac{y}{x} = \frac{-5}{-3} = \frac{5}{3}$$

The point $(-3, -5)$ is in the third quadrant, so add π to whatever the calculator gives for the inverse tangent of $\frac{5}{3}$:

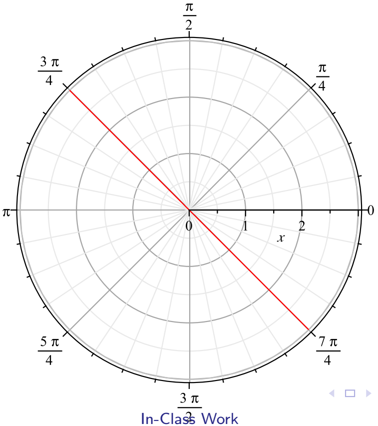
$$\theta = \pi + \arctan\left(\frac{5}{3}\right) \approx pi + 1.0304$$

Thus the rectangular point $(-3, -5)$ is (roughly) equivalent to the polar point $\left(\sqrt{34}, \pi + 1.0304\right)$.

Solutions

3. Sketch the graph of $\theta = 3\pi/4$.

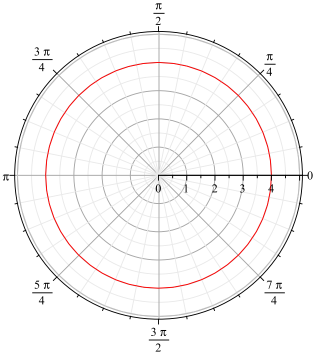
Because our angle is fixed but r (the distance from the origin) can be anything, this will just be a line radiating out from the origin at an angle of $3\pi/4$ in both directions.



Solutions

4. Sketch the graph of $r = 4$.

This will be a circle of radius 4. θ must go from 0 to 2π to sketch out the entire circle.



Solutions

5. Sketch the graph of $r = \sin(4\theta)$.

To sketch the graph, plot points.

θ	$r(\theta)$
0	0
$\pi/8$	1
$\pi/4$	0
$3\pi/8$	-1
$\pi/2$	0
$5\pi/8$	1
$3\pi/4$	0
$7\pi/8$	-1
π	0
$\vdots$	$\vdots$

For graph, see end of today's Maple file