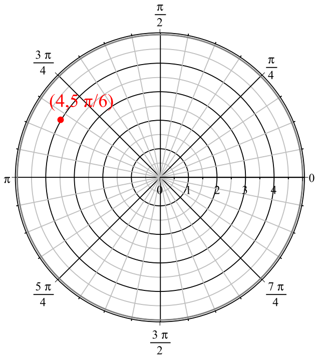
Where We've Been: Polar Coordinates

Any point in the plane can be specified by its distance r from the origin (r may be positive or negative) and the angle θ that the line connecting the point to the origin forms with the positive x-axis.



We can convert from polar to rectangular coordinates by using the conversion equations:

$$x = r\cos(\theta) \qquad y = r\sin(\theta)$$

$$\Rightarrow \quad x = 4\cos\left(\frac{5\pi}{6}\right) \qquad y = 4\sin\left(\frac{5\pi}{6}\right)$$

$$\Rightarrow \quad x = 4\cdot\frac{-\sqrt{3}}{2} \qquad y = 4\cdot\frac{1}{2}$$

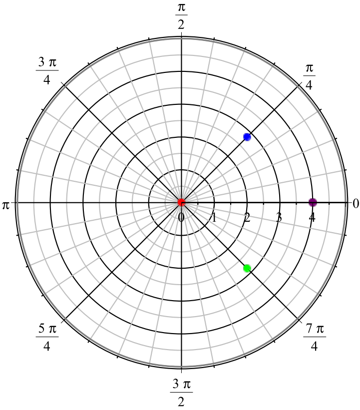
Thus the point with polar coordinates $\left(4, \frac{5\pi}{6}\right)$ has rectangular coordinates $\left(-2\sqrt{3}, 2\right)$.

Polar functions: Plotting points

We can graph a polar function simply by plotting points.

Graph the function $r = 4\cos(\theta)$.

θ	$r = f(\theta)$	$\big(f(\theta), \theta\big)$
0	$4\cos(0) = 4$	$(4, 0)$
$\frac{\pi}{4}$	$4\cos\left(\frac{\pi}{4}\right) = 2\sqrt{2}$	$\left(2\sqrt{2}, \frac{\pi}{4}\right)$
$\frac{\pi}{2}$	$4\cos\left(\frac{\pi}{2}\right) = 0$	$\left(0, \frac{\pi}{2}\right)$
$\frac{3\pi}{4}$	$4\cos\left(\frac{3\pi}{4}\right) = -2\sqrt{2}$	$\left(-2\sqrt{2}, \frac{3\pi}{4}\right)$
π	$4\cos(\pi) = -4$	$(-4, \pi)$
$\frac{5\pi}{4}$	$4\cos\left(\frac{5\pi}{4}\right) = -2\sqrt{2}$	$\left(-2\sqrt{2}, \frac{5\pi}{4}\right)$
$\frac{3\pi}{2}$	$4\cos\left(\frac{3\pi}{2}\right) = 0$	$\left(0, \frac{3\pi}{2}\right)$
$\frac{7\pi}{4}$	$4\cos\left(\frac{7\pi}{4}\right) = 2\sqrt{2}$	$\left(2\sqrt{2}, \frac{7\pi}{4}\right)$
2π	$4\cos(2\pi) = 4$	$(4, 2\pi)$



A circle is traced out for $0 \leq \theta \leq \pi$

In Class Work

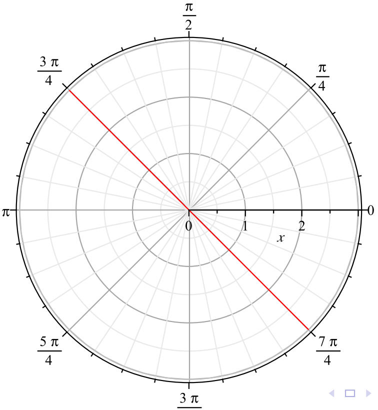
Sketch the graphs of the following polar equations (usually, you'll do this by plotting points).

1. $\theta = 3\pi/4$
2. $r = 4$
3. $r = \sin(4\theta)$
4. $r = 2\cos(3\theta) + 3$

Solutions

1. Sketch the graph of $\theta = 3\pi/4$.

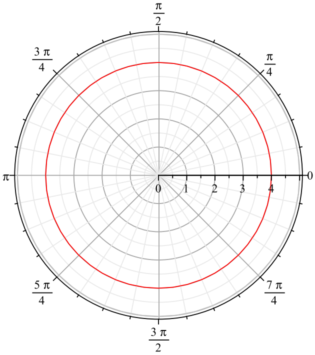
Because our angle is fixed but r (the distance from the origin) can be anything, this will just be a line radiating out from the origin at an angle of $3\pi/4$ in both directions.



Solutions

2. Sketch the graph of $r = 4$.

This will be a circle of radius 4. θ must go from 0 to 2π to sketch out the entire circle.



Solutions

3. Sketch the graph of $r = \sin(4\theta)$.

To sketch the graph, plot points.

θ	$r(\theta)$
0	0
$\pi/8$	1
$\pi/4$	0
$3\pi/8$	-1
$\pi/2$	0
$5\pi/8$	1
$3\pi/4$	0
$7\pi/8$	-1
π	0
$\vdots$	$\vdots$

For graph, see end of the Maple file from the last class

In Class Work

4. Sketch the graph of $r = 2\cos(3\theta) + 3$.

Again, plot points:

θ	$r(\theta)$
0	5
$\pi/6$	3
$2\pi/6$	1
$3\pi/6$	3
$4\pi/6$	5
$5\pi/6$	3
$6\pi/6$	1
$7\pi/6$	3
$8\pi/6$	5
$9\pi/6$	3
$10\pi/6$	1
$11\pi/6$	3