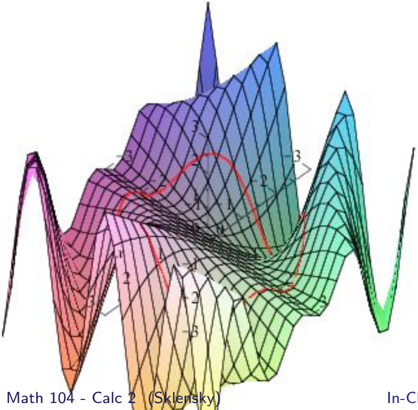
# Motivating the Chain Rule

Suppose

- $f(x, y) = x\cos(xy)$ models surface of a mountain above the point $(x, y)$
- $x(\theta) = 2\cos(\theta)$, $y(\theta) = 2\sin(\theta)$ models a circular path along the mountain

Then $f\left(x(\theta), y(\theta)\right)$ gives the altitude of any point on that circular path.



**Question:** How is the elevation changing at $\theta = \frac{\pi}{2}$?

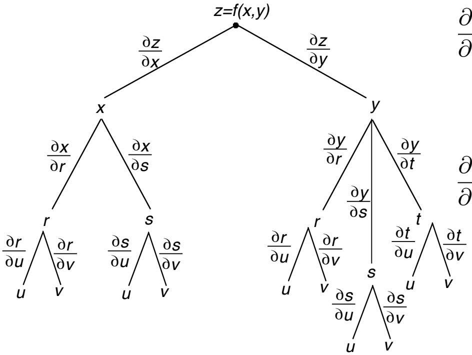
# In Class Work

1. Draw a tree diagram to figure out the chain rule for composite functions of the form $z = g(u, v) = f(x(r, s), y(r, s, t))$, where $r, s$, and $t$ are all functions of $u$ and $v$.

2. Suppose that $w = f(x, y, z)$ and that $x, y$, and $z$ are all functions of $r, s$, and $t$.
    (a) How many partial derivatives do you need to calculate, in order to determine $\frac{\partial w}{\partial t}$?
    (b) What *is* the expression for $\frac{\partial w}{\partial t}$?

# Solutions

1. Draw a tree diagram to figure out the chain rule for composite functions of the form $z = g(u, v) = f(x(r, s), y(r, s, t))$, where $r, s$, and $t$ are all functions of $u$ and $v$.

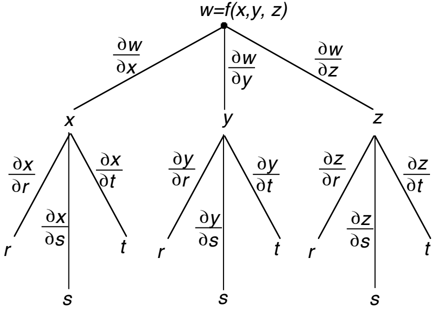



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial r}\frac{\partial r}{\partial u} + \frac{\partial x}{\partial s}\frac{\partial s}{\partial u}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial r}\frac{\partial r}{\partial u} + \frac{\partial y}{\partial s}\frac{\partial s}{\partial u} + \frac{\partial y}{\partial t}\frac{\partial t}{\partial u}\right)$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial r}\frac{\partial r}{\partial v} + \frac{\partial x}{\partial s}\frac{\partial s}{\partial v}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial r}\frac{\partial r}{\partial v} + \frac{\partial y}{\partial s}\frac{\partial s}{\partial v} + \frac{\partial y}{\partial t}\frac{\partial t}{\partial v}\right)$$

# Solutions

2.How many partial derivatives do you need to calculate, in order to determine $\frac{\partial w}{\partial t}$?




Just to calculate this one partial using the chain rule, I need to calculate six different partial derivatives.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$